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MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2018

## Advanced Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Find the ratio in which the point C, (9, 8, -10) divides the line segment joining the points A(5, 4, -6) and B(3, 2, -4). (06 Marks)
- b. If  $\cos \alpha$ ,  $\cos \beta$ ,  $\cos \gamma$  are the direction cosines of a straight line, prove that  
(i)  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$  (ii)  $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ . (07 Marks)
- c. Find the constant K such that the angle between the lines with direction ratios (-2, 1, -1) and (1, -K, 1) is  $90^\circ$ . (07 Marks)
- 2 a. Show that the angles between the diagonals of a cube is  $\theta = \cos^{-1}(1/3)$ . (06 Marks)
- b. Find the equation of the plane through the points (1, 0, -1) and (3, 2, 2) and parallel to the line  $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$ . (07 Marks)
- c. Show that the points A(-6, 3, 2), B(3, -2, 4), C(5, 7, 3) and D(-13, 17, -1) are coplanar. Also find the equation of the plane containing them. (07 Marks)
- 3 a. Find the angle between the vectors  $\vec{a} = 2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ ,  $\vec{b} = 12\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$ . (06 Marks)
- b. Find the area of a parallelogram whose adjacent sides are  $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  and  $2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ . (07 Marks)
- c. Find a unit vector perpendicular to both vectors  $\vec{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ ,  $\vec{b} = 7\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ . (07 Marks)
- 4 a. Show that the four points whose position vectors are  $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ ,  $6\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ ,  $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$  and  $2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$  are coplanar. (06 Marks)
- b. A particle moves along the curve  $x = t^3 + 1$ ,  $y = t^2$ ,  $z = 2t + 3$  where t is the time. Find the components of velocity and acceleration at  $t = 1$  in the direction of  $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ . (07 Marks)
- c. Find the directional derivative of  $f(x, y, z) = xy^2 + yz^3$  at the point (2, -1, 1) in the direction of vector  $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ . (07 Marks)
- 5 a. Find  $\text{div } F$  and  $\text{curl } F$  where  $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ . (06 Marks)
- b. Show that  $F = x(y-z)\mathbf{i} + y(z-x)\mathbf{j} + z(x-y)\mathbf{k}$  is solenoidal. (07 Marks)
- c. Find the constants a and b so that the vector  $\vec{F} = (axy + z^3)\hat{\mathbf{i}} + (3x^2 - z)\hat{\mathbf{j}} + (bxz^2 - y)\hat{\mathbf{k}}$  is irrotational. (07 Marks)
- 6 a. Find the Laplace transforms of  $1 + 2t^3 - 4e^{3t} + 5e^{-t}$ . (07 Marks)
- b. Find the Laplace transform of  $t^2 \sin^2 t$ . (07 Marks)
- c. Find the Laplace transform of  $\frac{\sin at}{t}$ . (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



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- 7 a. Find the inverse Laplace transform of  $\frac{3s-4}{16-s^2}$ . (06 Marks)
- b. Find the inverse Laplace transform of  $\frac{1}{s^2+4s+9}$ . (07 Marks)
- c. Evaluate  $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\}$ . (07 Marks)
- 8 a. Obtain the Laplace transforms of  $f'(t)$ ,  $f''(t)$ . (08 Marks)
- b. Solve the differential equation using Laplace transforms  $y'' - 3y' + 2y = 1 - e^{2t}$  under the conditions  $y(0) = 1$ ,  $y'(0) = 0$ . (12 Marks)

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