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MATDIP301

Third Semester B.E. Degree Examination, June/July 2019
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express square root of $1 - i$ in the form of $x + iy$. **(07 Marks)**

b. Find the modulus and amplitude of the following and express each in polar form.

(i) $1 - i\sqrt{3}$ (ii) $\dfrac{1-i}{1+i}$ **(07 Marks)**

c. Expand $\cos^6\theta$ in series of multiples of θ. **(06 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\dfrac{x}{(x+1)(x-2)}$. **(07 Marks)**

c. If $y = \log\left(x + \sqrt{1+x^2}\right)$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y = 0$. **(07 Marks)**

3 a. Find the angle between radius vector and the tangent of the curve $r = a(1 + \cos\theta)$. **(06 Marks)**

b. Find the Taylor's series expansion of the function e^x about $x = 1$. **(07 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\log_e(1 + x)$ up to third degree terms. **(07 Marks)**

4 a. If $\cos u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{2}\cot u$. **(06 Marks)**

b. If $x = r\cos\theta$ and $y = r\sin\theta$, prove that $JJ' = 1$. **(07 Marks)**

c. If $x^y + y^x = c$, where c is a constant, find $\dfrac{dy}{dx}$. **(07 Marks)**

5 a. Obtain the reduction formula $I_n = \int \sin^n x\, dx$, where n is a positive integer. **(06 Marks)**

b. Evaluate : $\int_0^1\int_0^{\sqrt{x}} xy(x+y)\,dx\,dy$ **(07 Marks)**

c. Evaluate : $\int_0^1\int_0^{1-z}\int_0^{1-z-y}(x+y+z)\,dx\,dy\,dz$ **(07 Marks)**

6 a. Prove the following :

$\beta(m, n) = \beta(n, m)$ **(06 Marks)**

b. Prove that $\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi}$ **(07 Marks)**

c. Using Gamma function, evaluate the integral $\int_0^1 \dfrac{1}{\sqrt{1-x^4}}\,dx$ **(07 Marks)**

7 a. Solve : $(x+y+1)^2\frac{dy}{dx}=1$ **(06 Marks)**

b. Solve : $\frac{dy}{dx}=1+x^2+y^2+x^2y^2$. **(07 Marks)**

c. Solve : $\left(x^2-xy+y^2\right)dx-xy\,dy=0$ **(07 Marks)**

8 Solve the following second order O.D.Es.

a. $\frac{d^2y}{dx^2}+y=e^x$ **(06 Marks)**

b. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\cos^2 x$ **(07 Marks)**

c. $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=2(1+x)$. **(07 Marks)**

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