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MATDIP301

Third Semester B.E. Degree Examination, Dec.2017/Jan.2018

Advanced Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Find the modulus and amplitude of $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Express the complex number $2 + 3i + \frac{1}{1-i}$ in the form a + ib. **(07 Marks)**

c. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4(\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3(\cos 5\theta + i\sin 5\theta)^{-4}}$. **(07 Marks)**

2 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x^2}{2x^2+7x+6}$. **(07 Marks)**

c. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)\,y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. **(07 Marks)**

3 a. If ϕ is the angle between the tangent and radius vector to the curve $r = f(\theta)$ at any point (r, θ), prove that $\tan\theta = \frac{rd\theta}{dr}$. **(06 Marks)**

b. Find the angle of intersection between the curves $r^n = a^n\cos n\theta$ and $r^n = b^n\sin n\theta$. **(07 Marks)**

c. Using Maclaurin's series, expand tan x up to the term containing x^5. **(07 Marks)**

4 a. If $Z = f(x + ct) + \phi(x - ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = C^2\frac{\partial^2 z}{\partial x^2}$. **(06 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ tan u. **(07 Marks)**

c. If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

PART – B

5 a. Obtain the reduction formula for $\int \cos^n x dx$. **(06 Marks)**

b. Using reduction formula evaluate $\int_0^a \frac{x^7}{\sqrt{a^2-x^2}}dx$. **(07 Marks)**

c. Evaluate $\int_0^1\int_0^1 e^{x+y}\,dx\,dy$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Evaluate $\int_{0}^{1}\int_{0}^{2}\int_{1}^{2} x^2yz\ dxdydz$. (07 Marks)

b. Prove that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

c. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (06 Marks)

7 a. Solve $3e^x \tan y\ dx + (1-e^x)\sec^2 y dy = 0$. (06 Marks)

b. Solve $(2x + 3y + 4)dx - (4x + 6y + 5)\ dy = 0$. (07 Marks)

c. Solve $\dfrac{dy}{dx} + y \tan x = \cos x$. (07 Marks)

8 a. Solve $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = -2 \cos hx$. (06 Marks)

b. Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$. (07 Marks)

c. Solve $\dfrac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$. (07 Marks)

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