



USN

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020 Additional Mathematics - I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express the following complex number in the form of
$$x + iy : \frac{(1+i)(1+3i)}{1+5i}$$
. (06 Marks)

b. Prove that
$$\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$$
. (07 Marks)

c. If
$$\overrightarrow{a} = (3,-1,4)$$
, $\overrightarrow{b} = (1,2,3)$ and $\overrightarrow{c} = (4,2,-1)$, find $\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c})$. (07 Marks)

2 a. Find the angle between the vectors,
$$\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$$
 and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. (06 Marks)

b. Prove that
$$\begin{bmatrix} \vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a} \end{bmatrix} = \begin{bmatrix} \vec{a}, \vec{b}, \vec{c} \end{bmatrix}^2$$
 (07 Marks)

c. Find the fourth roots of
$$-1+i\sqrt{3}$$
 and represent them on the argand diagram. (07 Marks)

3 a. Obtain the Maclaurin's expansion of
$$log_e(1+x)$$
. (06 Marks)

b. If
$$u = \sin^{-1} \left[\frac{x^3 + y^3}{x + y} \right]$$
, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$. (07 Marks)

c. If
$$u = x(1-y)$$
, $v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$. (07 Marks)

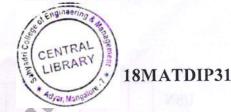
b. If
$$u = x^2 - 2y$$
; $v = x + y$ find $\frac{\partial(u, v)}{\partial(x, y)}$. (07 Marks)

c. If
$$u = f(x - y, y - z, z - x)$$
, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (07 Marks)

5 a. Find the velocity and acceleration of a particle moves along the curve,
$$\vec{r} = e^{-2t}\hat{i} + 2\cos 5t\hat{j} + 5\sin 2t\hat{k} \text{ at any time t.}$$
 (06 Marks)

b. Find div
$$\vec{F}$$
 and curl \vec{F} , where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)

c. Show that
$$\vec{F} = (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2xz)\hat{k}$$
 is conservative force field and find the scalar potential. (07 Marks)



OR

a. Show that the vector field, $\vec{F} = (3x + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal.

b. Find the directional derivative of $\phi = \frac{xz}{x^2 + y^2}$ at (1, -1, 1) in the direction of $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$.

Find the constant 'a' such that the vector field $\vec{F} = 2xy^2z^2\hat{i} + 2x^2yz^2\hat{j} + ax^2y^2z\hat{k}$ is (07 Marks) irrotational.

Module-4

a. Find the reduction formula for \sin \(^n\) xdx. (06 Marks)

b. Evaluate $\int_{0}^{1} \int_{0}^{3} x^3 y^3 dx dy$. (07 Marks)

c. Evaluate $\int_{0}^{3} \int_{0}^{2} \int_{0}^{1} (x+y+z) dz dx dy$. (07 Marks)

a. Evaluate: $\int \sin^6(3x) dx$ (06 Marks)

b. Evaluate : $\int_{0}^{1} \int_{x}^{\sqrt{x}} xy \, dy dx$. (07 Marks)

c. Evaluate: $\int_{0}^{1} \int_{0}^{1-x} \int_{0}^{1-x-y} xyzdzdydx$. (07 Marks)

Module-5

a. Solve: $\frac{dy}{dx} + y \cot x = \sin x$. (06 Marks)

b. Solve: $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. c. Solve: $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. (07 Marks)

(07 Marks)

a. Solve: $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (06 Marks)

b. Solve: $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y.$ (07 Marks)

c. Solve: $[1+(x+y)\tan y]\frac{dy}{dx}+1=0$. (07 Marks)