CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Express the following complex number in the form of $x+iy$: $\dfrac{(1+i)(1+3i)}{1+5i}$. **(06 Marks)**

b. Prove that $\left(\dfrac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4=\cos 8\theta+i\sin 8\theta$. **(07 Marks)**

c. If $\vec{a}=(3,-1,4)$, $\vec{b}=(1,2,3)$ and $\vec{c}=(4,2,-1)$, find $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$. **(07 Marks)**

OR

2 a. Find the angle between the vectors, $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(06 Marks)**

b. Prove that $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$ **(07 Marks)**

c. Find the fourth roots of $-1+i\sqrt{3}$ and represent them on the argand diagram. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's expansion of $\log_e(1+x)$. **(06 Marks)**

b. If $u=\sin^{-1}\left[\dfrac{x^3+y^3}{x+y}\right]$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=2\tan u$. **(07 Marks)**

c. If $u=x(1-y)$, $v=xy$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

OR

4 a. Obtain the Maclauvin's series expansion of the function $\log_e \sec x$. **(06 Marks)**

b. If $u=x^2-2y;\ v=x+y$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$. **(07 Marks)**

c. If $u=f(x-y,y-z,z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(07 Marks)**

Module-3

5 a. Find the velocity and acceleration of a particle moves along the curve, $\vec{r}=e^{-2t}\hat{i}+2\cos 5t\hat{j}+5\sin 2t\hat{k}$ at any time t. **(06 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F}=\nabla\left(x^3+y^3+z^3-3xyz\right)$. **(07 Marks)**

c. Show that $\vec{F}=(2xy+z^2)\hat{i}+\left(x^2+2yz\right)\hat{j}+\left(y^2+2xz\right)\hat{k}$ is conservative force field and find the scalar potential. **(07 Marks)**

OR

6 a. Show that the vector field, $\vec{F} = (3x+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}$ is solenoidal. **(06 Marks)**

b. Find the directional derivative of $\phi = \frac{xz}{x^2+y^2}$ at (1, −1, 1) in the direction of $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$. **(07 Marks)**

c. Find the constant 'a' such that the vector field $\vec{F} = 2xy^2z^2\hat{i} + 2x^2yz^2\hat{j} + ax^2y^2z\hat{k}$ is irrotational. **(07 Marks)**

Module-4

7 a. Find the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^3 x^3y^3\,dxdy$. **(07 Marks)**

c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z)\,dzdxdy$. **(07 Marks)**

OR

8 a. Evaluate : $\int_0^{\frac{\pi}{6}} \sin^6(3x)\,dx$. **(06 Marks)**

b. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} xy\,dydx$. **(07 Marks)**

c. Evaluate : $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\,dzdydx$. **(07 Marks)**

Module-5

9 a. Solve : $\frac{dy}{dx} + y\cot x = \sin x$. **(06 Marks)**

b. Solve : $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. **(07 Marks)**

c. Solve : $3x(x+y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$. **(07 Marks)**

OR

10 a. Solve : $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. **(06 Marks)**

b. Solve : $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(07 Marks)**

c. Solve : $[1 + (x+y)\tan y]\frac{dy}{dx} + 1 = 0$. **(07 Marks)**

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