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18MAT31

Third Semester B.E. Degree Examination, Feb./Mar. 2022 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Evaluate (i) $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$ (ii) $L(t^2 e^{-3t} \sin 2t)$ (06 Marks)
- b. If $f(t) = \begin{cases} t, & 0 \leq t \leq a \\ 2a - t, & a \leq t \leq 2a \end{cases}$, $f(t + 2a) = f(t)$ then show that $L(f(t)) = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$ (07 Marks)
- c. Solve by using Laplace Transforms
 $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$, $y(0) = 0$, $y'(0) = 0$ (07 Marks)

OR

- 2 a. Evaluate $L^{-1}\left(\frac{4s+5}{(s+1)^2(s+2)}\right)$ (06 Marks)
- b. Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ by using convolution theorem. (07 Marks)
- c. Express $f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ \sin 2t, & \pi \leq t < 2\pi \\ \sin 3t, & t \geq 2\pi \end{cases}$
 in terms of unit step function and hence find its Laplace Transform. (07 Marks)

Module-2

- 3 a. Obtain fourier series for the function $f(x) = |x|$ in $(-\pi, \pi)$ (06 Marks)
- b. Expand $f(x) = \frac{(\pi-x)^2}{4}$ as a Fourier series in the interval $(0, 2\pi)$ and hence deduce that
 $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ (07 Marks)
- c. Express y as a Fourier series upto the second harmonic given :

x:	0	60	120	180	240	300
y:	4	3	2	4	5	6

(07 Marks)

OR

- 4 a. Find the Half-Range sine series of $\pi x - x^2$ in the interval $(0, \pi)$ (06 Marks)
- b. Obtain fourier expansion of the function $f(x) = 2x - x^2$ in the interval $(0, 3)$. (07 Marks)



- c. Obtain the Fourier expansion of y upto the first harmonic given :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

Module-3

- 5 a. If $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$, find the Fourier transform of $f(x)$ and hence find the value of $\int_0^{\infty} \frac{\sin x}{x} dx$ (06 Marks)
- b. Find the infinite Fourier cosine transform of $e^{-\alpha x}$. (07 Marks)
- c. Solve using z-transform $y_{n+2} - 4y_n = 0$ given that $y_0 = 0, y_1 = 2$ (07 Marks)

OR

- 6 a. Find the fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$; $m > 0$. (06 Marks)
- b. Obtain the z-transform of $\cos n\theta$ and $\sin n\theta$. (07 Marks)
- c. Find the inverse z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ (07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} = x^3 + y$, $y(1) = 1$ using Taylor's series method considering up to fourth degree terms and find $y(1.1)$. (06 Marks)
- b. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ compute $y(0.2)$ by taking $h = 0.2$ using Runge - Kutta method of fourth order. (07 Marks)
- c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find $y(0.4)$ correct to 4 decimal places using Adams-Bashforth method. (07 Marks)

OR

- 8 a. Use fourth order Runge-Kutta method, to find $y(0.8)$ with $h = 0.4$, given $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$ (06 Marks)
- b. Use modified Euler's method to compute $y(20.2)$ and $y(20.4)$ given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ Taking $h = 0.2$. (07 Marks)
- c. Apply Milne's predictor-corrector formulae to compute $y(2.0)$ given $\frac{dy}{dx} = \frac{x+y}{2}$ with

x	0.0	0.5	1.0	1.5
y	2.000	2.6360	3.5950	4.9680

(07 Marks)

**Module-5**

- 9 a. Using Runge-Kutta method, solve

$$\frac{d^2y}{dx^2} = x \left(\frac{dy}{dx} \right)^2 - y^2, \text{ for } x = 0.2, \text{ correct to four decimal places, using initial conditions } y(0) = 1, y'(0) = 0$$

(07 Marks)

- b. Derive Euler's equation in the standard form viz,
- $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$

(07 Marks)

- c. Find the extremal of the functional
- $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$

(06 Marks)

OR

- 10 a. Given the differential equation
- $2 \frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$
- and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	2.0657

Compute $y(1.4)$ by applying Milne's Predictor-corrector formula.

(07 Marks)

- b. Prove that geodesics of a plane surface are straight lines.

(07 Marks)

- c. On what curves can the functional
- $\int_0^1 (y'^2 + 12xy) dx$
- with
- $y(0) = 0, y(1) = 1$
- can be extremized?

(06 Marks)

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