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18MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2021
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ **(06 Marks)**

b. Find the radius of curvature for the parabola $\frac{2a}{r} = 1 + \cos\theta$ **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$ **(08 Marks)**

OR

2 a. Find the angle of intersection of the curves $r = 2\sin\theta$ and $r = 2\cos\theta$ **(06 Marks)**

b. Find the pedal equation of the curve $r^m = a^m [\cos m\theta + \sin m\theta]$ **(06 Marks)**

c. For the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that

$$\sqrt{1+\cos 2x} = \sqrt{2}\left[1 - \frac{x^2}{2} + \frac{x^4}{24} - \ldots\ldots\right]$$ **(06 Marks)**

b. Evaluate i) $\underset{x \to 0}{\text{Lt}}\left(\frac{1}{x}\right)^{2\sin x}$ ii) $\underset{x \to 0}{\text{Lt}}\left[\frac{a^x + b^x + c^x}{3}\right]^{\frac{1}{x}}$ **(07 Marks)**

c. Examine the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ for its extreme values. **(07 Marks)**

OR

4 a. If $u = f(y-z, z-x, x-y)$ then prove that $u_x + u_y + u_z = 0$. **(06 Marks)**

b. If $u = 3x + 2y - z$; $v = x - 2y + z$; $w = x^2 + 2xy - xz$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$ **(07 Marks)**

c. The pressure P at any point (x, y, z) in space $P = 400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$. **(07 Marks)**

Module-3

5 a. Evaluate: $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$ **(06 Marks)**

b. Obtain the relation between Beta and Gama functions in the form $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$ **(07 Marks)**

c. Find the centre of Gravity of the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

OR

6 a. Change the order of integration and evaluate $\int_0^1 \int_{\sqrt{y}}^1 dx\,dy$ **(06 Marks)**

b. A Pyramid is bounded by three coordinate planes and the plane x + 2y + 3z = 6. Compute the volume by double integration. **(07 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

Module-4

7 a. Solve $\left[y\left(x+\frac{1}{x}\right)+\cos y\right]dx + \left[x+\log x - x\sin y\right]dy$ **(06 Marks)**

b. A body in air at 25°C cools from 100°C to 75°C in 1 minute, find the temperature of the body at the end of 3 minutes. **(07 Marks)**

c. Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Solve: $xyp^2 - (x^2 + y^2)p + xy = 0$ **(06 Marks)**

b. Solve: $\frac{dy}{dx} + y\tan x = y^3 \sec x$ **(07 Marks)**

c. Solve the equation $L\frac{di}{dt} + Ri = E_o \sin wt$ where L, R and E_o are constants and discuss the case when t increases indefinitely. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ using elementary row operation. **(06 Marks)**

b. Find largest eigen value and eigen vector of the matrix $\begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}$ by taking $(1, 0, 0)^T$ as initial eigen vector by Rayleigh's power method (perform 6 iteration). **(07 Marks)**

c. Solve the system of equations x + y + z = 9 ; x - 2y + 3z = 8 ; 2x + y – z = 3, by Gauss Jordan method. **(07 Marks)**

OR

10 a. For what value of λ and μ the system of equations x + y + z = 6 ; x + 2y + 3z = 10 ; x + 2y + λz = μ has i) No solution ii) Unique solution iii) Infinite number of solution. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$ into the diagonal form. **(07 Marks)**

c. Solve the system of equations 83x + 11y – 4z = 95, 7x + 52y + 13z = 104 , 3x + 8y + 29z = 71 by Gauss Seidal method (carry out 4 iteration). **(07 Marks)**

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