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18MAT11

## First Semester B.E. Degree Examination, Dec.2019/Jan.2020 Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. With usual notations prove that  $\tan \phi = r \left( \frac{d\theta}{dr} \right)$ . (06 Marks)
- b. Find the angle between the curves  $r = \sin\theta + \cos\theta$  and  $r = 2 \sin\theta$  (06 Marks)
- c. Show that the radius of curvature for the catenary of uniform strength  $y = a \log \sec \left( \frac{x}{a} \right)$  is  $a \sec \left( \frac{x}{a} \right)$ . (08 Marks)

OR

- 2 a. Show that the pairs of curves  $r = a(1 + \cos\theta)$  and  $r = b(1 - \cos\theta)$  intersect each other Orthogonally. (06 Marks)
- b. Find the pedal equation of the curve  $r^n = a^n \cos n\theta$ . (06 Marks)
- c. Show that the evolute of  $y^2 = 4ax$  is  $27ay^2 = 4(x + a)^3$  (08 Marks)

### Module-2

- 3 a. Find the Macluarin's series for  $\tan x$  upto the term  $x^4$ . (06 Marks)
- b. Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{a^x + b^x + c^x}{3} \right]^{1/x}$  (07 Marks)
- c. If  $U = f(x-y, y-z, z-x)$ , prove that  $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$  (07 Marks)

OR

- 4 a. Expand  $\log (\sec x)$  upto the term containing  $x^4$  using Maclaurin's series. (06 Marks)
- b. Find the extreme values of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ . (07 Marks)
- c. Find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  where  $u = x^2 + y^2 + z^2, v = xy + yz + zx, w = x + y + z$ . (07 Marks)

### Module-3

- 5 a. Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dzdydx$  (06 Marks)
- b. Evaluate  $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} (2-x)dydx$  by changing the order of integration. (07 Marks)
- c. Prove that  $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$  (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



OR

- 6 a. Evaluate  $\int \int y \, dx \, dy$  over the region bounded by the first quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . (06 Marks)
- b. Find by double integration the area enclosed by the curve  $r = a(1 + \cos\theta)$  between  $\theta = 0$  and  $\theta = \pi$ . (07 Marks)
- c. Show that  $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta = \pi$ . (07 Marks)

**Module-4**

- 7 a. Solve  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$  (06 Marks)
- b. Solve  $r \sin\theta - \cos\theta \frac{dr}{d\theta} = r^2$  (07 Marks)
- c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation  $L \frac{di}{dt} + Ri = E$ , where L and R are constants and initially the current i is zero. Find the current at any time t. (07 Marks)

OR

- 8 a. Solve  $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$ . (06 Marks)
- b. Find the orthogonal trajectories of the family of parabolas  $y^2 = 4ax$ . (07 Marks)
- c. Solve  $p^2 + 2py \cot x = y^2$ . (07 Marks)

**Module-5**

- 9 a. Find the rank of  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$  by elementary row transformations. (06 Marks)
- b. Apply Gauss-Jordan method to solve the system of equations  $2x_1 + x_2 + 3x_3 = 1$ ,  $4x_1 + 4x_2 + 7x_3 = 1$ ,  $2x_1 + 5x_2 + 9x_3 = 3$ . (07 Marks)
- c. Find the largest Eigen value and the corresponding Eigen vector of the matrix  $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$  by power method. Using initial vector  $(100)^T$ . (07 Marks)

OR

- 10 a. Solve by Gauss elimination method  $x - 2y + 3z = 2$ ,  $3x - y + 4z = 4$ ,  $2x + y - 2z = 5$  (06 Marks)
- b. Solve the system of equations by Gauss-Seidal method  $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$ . (07 Marks)
- c. Reduce the matrix  $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$  to the diagonal form. (07 Marks)

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