CBCS SCHEME

USN 4SF18ME421

17MATDIP31

Third Semester B.E. Degree Examination, June/July 2019
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. (08 Marks)

b. Express the complex number $\frac{(1+i)(1+3i)}{1+5i}$ in the form a + ib. (06 Marks)

c. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$. (06 Marks)

OR

2 a. Show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cdot\cos^n\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{n\theta}{2}\right)$. (08 Marks)

b. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = 8\hat{i} - 4\hat{j} + \hat{k}$, then prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a} \times \vec{b}|$. (06 Marks)

c. Determine λ such that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar. (06 Marks)

Module-2

3 a. If $y = \cos(m\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$. (08 Marks)

b. Find the angle of intersection of the curves $r^2\sin 2\theta = a^2$ and $r^2\cos 2\theta = b^2$. (06 Marks)

c. Find the pedal equation of the curve $r = a(1+\sin\theta)$. (06 Marks)

OR

4 a. Obtain the Maclaurin's series expansion of $\log\sec x$ up to the terms containing x^6. (08 Marks)

b. If $u = \operatorname{cosec}^{-1}\left(\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right)$, prove that $xu_x + yu_y = -\frac{1}{6}\tan u$. (06 Marks)

c. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x+y+z$, $v = y+z$, $w = z$. (06 Marks)

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, $(n > 0)$. (08 Marks)

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. (06 Marks)

c. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ (06 Marks)

OR

6 a. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$. **(08 Marks)**

b. Evaluate $\int_0^\infty \frac{x^6}{(1+x^2)^{9/2}}\,dx$. **(06 Marks)**

c. Evaluate $\iint_A xy\,dx\,dy$ where A is the area bounded by the circle $x^2+y^2=a^2$ in the first quadrant. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$. Find the components of velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$. **(08 Marks)**

b. Find divergence and curl of the vector $\vec{F} = (xyz + y^2z)\,\hat{i} + (3x^2 + y^2z)\,\hat{j} + (xz^2 - y^2z)\,\hat{k}$. **(06 Marks)**

c. Find the directional derivative of $\phi = x^2yz^3$ at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at $t = 2$ and $t = 3$. **(08 Marks)**

b. Find $\text{curl}(\text{curl}\,\vec{A})$ where $\vec{A} = xy\,\hat{i} + y^2z\,\hat{j} + z^2y\,\hat{k}$. **(06 Marks)**

c. Find the constants a, b, c such that the vector field $(\sin y + az)\,\hat{i} + (bx\cos y + z)\,\hat{j} + (x + cy)\,\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$. **(08 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \sin x$. **(06 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. **(06 Marks)**

OR

10 a. Solve $x^2y\,dx - (x^3 + y^3)\,dy = 0$. **(08 Marks)**

b. Solve $x^2\frac{dy}{dx} = 3x^2 - 2xy + 1$. **(06 Marks)**

c. Solve $\left[y\left(1 + \frac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]\,dy = 0$. **(06 Marks)**

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