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17MAT31

## Third Semester B.E. Degree Examination, Dec.2018/Jan.2019 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

### Module-1

- 1 a. Find the Fourier series expansion for the periodic function  $f(x)$ , if in one second

$$f(x) = \begin{cases} 0; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases}$$

(08 Marks)

- b. Expand the function  $f(x) = x(\pi-x)$  over the interval  $(0, \pi)$  in half range Fourier cosine series. (06 Marks)
- c. The following value of function  $y$  gives the displacement in inches of a certain machine part for rotations  $x$  of a flywheel. Expand  $y$ -in terms of Fourier series upto the second harmonic.

Rotations	$x$	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	$\pi$
Displacement	$y$	0	9.2	14.4	17.8	17.3	11.7	0

(06 Marks)

**OR**

- 2 a. Find the Fourier series expansion for the function :

$$f(x) = \begin{cases} \pi x; & 0 \leq x \leq 1 \\ \pi(2-x); & 1 \leq x \leq 2 \end{cases}$$

and deduce  $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

(08 Marks)

- b. Expand in Fourier series  $f(x) = (\pi - x)^2$  over the interval  $0 \leq x \leq 2\pi$ . (06 Marks)
- c. The following table gives the variations of periodic current over a period  $T$ .

$t$ (secs)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	$T$
A (Amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Expand the function (periodic current) by Fourier series and show that there is a direct current part of 0.75 amp and also obtain amplitude of first harmonic. (06 Marks)

### Module-2

- 3 a. Find Fourier transform of  $f(x) = \begin{cases} 1-x^2; & |x| < 1 \\ 0; & |x| > 1 \end{cases}$

and hence evaluate  $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$ .

(08 Marks)

- b. Find Fourier Cosine transform of the function :

$$f(x) = \begin{cases} 4x; & 0 < x < 1 \\ 4-x; & 1 < x < 4 \\ 0; & x > 4 \end{cases}$$

(06 Marks)

- c. Find z-transforms of : i)  $a^n \sin n\theta$  ii)  $a^{-n} \cos n\theta$ .

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.



OR

- 4 a. Find Fourier sine transform of  $f(x) = e^{-|x|}$  and hence evaluate :  $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx, m > 0$ . (08 Marks)
- b. Find z-transform of  $u_n = \cos h\left(\frac{n\pi}{2} + \theta\right)$ . (06 Marks)
- c. Solve the difference equation using z-transforms  $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ . Given  $u_0 = u_1 = 0$ . (06 Marks)

**Module-3**

- 5 a. If  $\theta$  - is the acute angle between the two regression lines relating the variables  $x$  and  $y$ , show that  $\text{Tan}\theta = \left(\frac{1-r^2}{r}\right) \left(\frac{\sigma_x \sigma_y}{\sigma_x^2 \sigma_y^2}\right)$ . (08 Marks)

Indicate the significance of the cases  $r = \pm 1$  and  $r = 0$ .

- b. Fit a straight line  $y = ax + b$  for the data.

x	12	15	21	25
y	50	70	100	120

- c. Find a real root of the equation by using Newton-Raphson method near  $x = 0.5$ ,  $xe^x = 2$ , perform three iterations. (06 Marks)

OR

- 6 a. Compute the coefficient of correlation and equation of regression of lines for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(08 Marks)

- b. The Growth of an organism after  $x$  - hours is given in the following table :

x (hours)	5	15	20	30	35	40
y (Growth)	10	14	25	40	50	62

Find the best values of  $a$  and  $b$  in the formula  $y = ae^{bx}$  to fit this data. (06 Marks)

- c. Find a real root of the equation  $\cos x = 3x - 1$  correct to three decimals by using Regula - False position method, given that root lies in between 0.6 and 0.7. Perform three iterations. (06 Marks)

**Module-4**

- 7 a. Find  $y(8)$  from  $y(1) = 24$ ,  $y(3) = 120$ ,  $y(5) = 336$ ,  $y(7) = 720$  by using Newton's backward difference interpolation formula. (08 Marks)
- b. Define  $f(x)$  - as a polynomial in  $x$  for the following data using Newton's divided difference formula. (06 Marks)

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

- c. Evaluate the integral  $I = \int_0^6 \frac{dx}{4x+5}$  using Simpson's  $\frac{1}{3}$ rd rule using 7 ordinates. (06 Marks)



OR

- 8 a. For the following data calculate the differences and obtain backward difference interpolation polynomial. Hence find  $f(0.35)$ . (08 Marks)

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.0	2.28

- b. Using Lagrange's interpolation find y when  $x = 10$ .

x	5	6	9	11
y	12	13	14	16

- c. Evaluate  $\int_0^1 \frac{x}{1+x^2} dx$  by Weddle's rule considering seven ordinates. (06 Marks)

### Module-5

- 9 a. Verify the Green's theorem in the plane for  $\int_C (x^2 + y^2)dx + 3x^2y dy$  where  $C$  - is the circle  $x^2 + y^2 = 4$  traced in positive sense. (08 Marks)
- b. Evaluate  $\int_C (\sin z dx - \cos x dy + \sin y dz)$  by using Stokes theorem, where  $C$  - is the boundary of the rectangle  $0 \leq x \leq \pi$ ,  $0 \leq y \leq 1$  and  $z = 3$ . (06 Marks)
- c. Find the curve on which the functional  $\int_0^1 [y'^2 + 12xy] dx$  with  $y(0) = 0$ ,  $y(1) = 1$  can be extremised. (06 Marks)

OR

- 10 a. Given  $f = (3x^2 - y)i + xzj + (yz - x)k$  evaluate  $\int_C f \cdot dr$  from  $(0, 0, 0)$  to  $(1, 1, 1)$  along the paths  $x = t$ ,  $y = t^2$  and  $z = t^3$ . (08 Marks)
- b. Derive Euler's equation in the form  $\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0$ . (06 Marks)
- c. Prove that the shortest distance between two points in a plane is a straight line. (06 Marks)

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