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17MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the n^{th} derivative of $\sin 2x \cos x$. (06 Marks)
- b. Prove that the following curves cuts orthogonally $r = a(1 + \sin \theta)$ and $r = a(1 - \sin \theta)$. (07 Marks)
- c. Find the radius of the curvature of the curve $r = a \sin n\theta$ at the pole. (07 Marks)

OR

- 2 a. If $\tan y = x$, prove that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. (06 Marks)
- b. With usual notations, prove that $\tan \phi = \frac{r d\theta}{dr}$. (07 Marks)
- c. Find the radius of curvature for the curve $n^2y = a(x^2 + y^2)$ at $(-2a, 2a)$. (07 Marks)

Module-2

- 3 a. Using Maclaurin's series prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$ (06 Marks)
- b. If $U = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} = -\frac{1}{4} \sin 2U$. (07 Marks)
- c. Find the Jacobian of $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. (07 Marks)

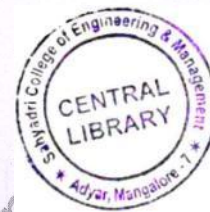
OR

- 4 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x}\right)^{1/x}$. (06 Marks)
- b. Find the Taylor's sense of $\log(\cos x)$ about the point $x = \frac{\pi}{3}$ upto the third degree. (07 Marks)
- c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (07 Marks)

Module-3

- 5 a. If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of a curve then, find velocity and acceleration at $t = 1$. (06 Marks)
- b. Find the constants a and b such that $\vec{F} = (axy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (bxz^2 - y)\mathbf{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
- c. Prove that $\text{div}(\text{curl } \vec{A}) = 0$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.



OR

- 6 a. Find the component of velocity and acceleration for the curve $\vec{r} = 2t^2\mathbf{i} + (t^2 - 4t)\mathbf{j} + (3t - 5)\mathbf{k}$ at the points $t = 1$ in the direction of $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. (06 Marks)
- b. If $\vec{t} = \nabla(xy^3z^2)$, find $\text{div } \vec{t}$ and $\text{curl } \vec{t}$ at the point $(1, -1, 1)$. (07 Marks)
- c. Prove that $\text{curl}(\text{grad } \phi) = 0$. (07 Marks)

Module-4

- 7 a. Prove that $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} dx = 3\pi$ using reduction formula. (06 Marks)
- b. Solve $(x^2 + y^2 + x)dx + xydy = 0$. (07 Marks)
- c. Find the orthogonal trajectory of $r^n = a \sin n\theta$. (07 Marks)

OR

- 8 a. Find the reduction formula for $\int \cos^n x dy$ and hence evaluate $\int_0^{\pi/2} \cos^n x dx$. (06 Marks)
- b. Solve $ye^{xy} dx + (xe^{xy} + 2y)dy = 0$. (07 Marks)
- c. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ by reducing to row echelon form. (06 Marks)
- b. Find the largest eigen and the corresponding eigen vector for $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation as $[1, 0.8, -0.8]^T$ by using power method. Carry out four iterations. (07 Marks)
- c. Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular. Find the inverse transformation. (07 Marks)

OR

- 10 a. Solve the equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$ by using Gauss Seidal method. Carry out three iterations taking the initial approximation to the solution as $(1, 0, 3)$. (06 Marks)
- b. Diagonalize the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. (07 Marks)
- c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation. (07 Marks)
