CBCS Scheme

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15MAT41

Fourth Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: 1. *Answer any FIVE full questions, choosing one full question from each module.*
2. *Use of statistical tables is permitted.*

Module-1

1 a. Use Taylor's series method to find y at x = 1.1, considering terms upto third degree given that $\frac{dy}{dx} = x + y$ and $y(1) = 0$. **(05 Marks)**

b. Using Runge-Kutta method, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$; y(0) = 1, taking h = 0.2. **(05 Marks)**

c. Given $\frac{dy}{dx} = x^2 - y$, y(0) = 1 and the values y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918, evaluate y(0.4), using Adams-Bashforth method. **(06 Marks)**

OR

2 a. Using Euler's modified method, find y(0.1) given $\frac{dy}{dx} = x - y^2$, y(0) = 1, taking h = 0.1. **(05 Marks)**

b. Solve $\frac{dy}{dx} = xy$; y(1) = 2, find the approximate solution at x = 1.2, using Runge-Kutta method. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ with the following data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, compute y at x = 0.8, using Milne's method. **(06 Marks)**

Module-2

3 a. Using Runge-Kutta method of order four, solve $y'' = y + xy'$, y(0) = 1, y′(0) = 0 to find y(0.2). **(05 Marks)**

b. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\, dx = 0$, if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Given $y'' = 1 + y'$; y(0) = 1, y′(0) = 1, compute y(0.4) for the following data, using Milne's predictor-corrector method.
y(0.1) = 1.1103 y(0.2) = 1.2427 y(0.3) = 1.399
y′(0.1) = 1.2103 y′(0.2) = 1.4427 y′(0.3) = 1.699. **(05 Marks)**

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(05 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. **(06 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in polar form. (05 Marks)

b. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$, using Cauchy's residue theorem. (05 Marks)

c. Find the bilinear transformation which maps $z = \infty, i, 0$ on to $w = 0, i, \infty$. (06 Marks)

OR

6 a. State and prove Cauchy's integral formula. (05 Marks)

b. If $u = \frac{\sin 2x}{\cosh 2y + \cos 2x}$, find the corresponding analytic function $f(z) = u + iv$. (05 Marks)

c. Discuss the transformation $w = z^2$. (06 Marks)

Module-4

7 a. Derive mean and standard deviation of the binomial distribution. (05 Marks)

b. If the probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individual (i) exactly 3 (ii) more than 2 individuals will suffer a bad reaction. (05 Marks)

c. The joint probability distribution for two random variables X and Y is as follows:

X \ Y	-3	-2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Determine: i) Marginal distribution of X and Y ii) Covariance of X and Y iii) Correlation of X and Y (06 Marks)

OR

8 a. Derive mean and standard deviation of exponential distribution. (05 Marks)

b. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. Given $P(0 < z < 1.2263) = 0.39$ and $P(0 < z < 1.14757) = 0.43$. (05 Marks)

c. The joint probability distribution of two random variables X and Y is as follows:

Y \ X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute: i) E(X) and E(Y) ii) E(XY) iii) COV(X, Y) iv) $\rho(X, Y)$ (06 Marks)

Module-5

9 a. Explain the terms: i) Null hypothesis ii) Type I and Type II errors. (05 Marks)

b. The nine items of a sample have the values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? (05 Marks)

c. Given the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ then show that A is a regular stochastic matrix. (06 Marks)

OR

10 a. A die was thrown 9000 times and of these 3220 yielded a 3 or 4, can the die be regarded as unbiased? (05 Marks)

b. Explain: i) Transient state ii) Absorbing state iii) Recurrent state (05 Marks)

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run, how often does he study? (06 Marks)