

# CBCS SCHEME



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15MAT11

## First Semester B.E. Degree Examination, June/July 2018 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

### Module-1

- 1 a. Find the  $n^{\text{th}}$  derivative of the  $\sin^3 x \cos^2 x$ . (06 Marks)  
b. Find angle between the pair of curves  $r = 6 \cos \theta$  and  $r = 2(1 + \cos \theta)$ . (05 Marks)  
c. Show that for the curve  $r(1 - \cos \theta) = 2a$  the radius of curvature is  $\frac{2}{\sqrt{a}} r^{3/2}$ . (05 Marks)

OR

- 2 a. Show that  $\left(\frac{2\rho}{a}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$  for the curve  $y = \frac{ax}{a+x}$ . (06 Marks)  
b. Find the Pedal equation of the curve  $r^m = a^m (\cos m\theta + \sin m\theta)$ . (05 Marks)  
c. If  $y = \log(x + \sqrt{1+x^2})$  prove that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$ . (05 Marks)

### Module-2

- 3 a. Expand  $\text{Log}(1 + \cos x)$  by Maclaurin's series upto the term containing  $x^4$ . (06 Marks)  
b. Evaluate  $\lim_{x \rightarrow \pi/2} (\sin x)^{\tan x}$ . (05 Marks)  
c. If  $u = \frac{yz}{x}$ ,  $v = \frac{zx}{y}$ ,  $w = \frac{xy}{z}$  show that  $\frac{\partial(u, v, w)}{\partial(x, y, z)} = u$  (05 Marks)

OR

- 4 a. If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$  show that  $xu_x + yu_y = \sin 2u$ . (06 Marks)  
b. If  $z = f(x, y)$ , where  $x = r \cos \theta$ ,  $y = r \sin \theta$  show that  $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$  (05 Marks)  
c. Expand  $\tan x$  in Taylor's series upto three in powers of  $\left(x - \frac{\pi}{4}\right)$ . (05 Marks)

### Module-3

- 5 a. A particle moves along the curve  $x = 1 - t^3$ ,  $y = 1 + t^2$  and  $z = 2t - 5$ , determine velocity and acceleration at  $t = 1$ . Also find the components of velocity and acceleration in the direction  $2i + j + 2k$ . (06 Marks)  
b. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at  $(2, -1, 2)$ . (05 Marks)  
c. Prove that  $\text{Div}(\phi \vec{A}) = \phi(\text{div} \vec{A}) + \text{grad} \phi \cdot \vec{A}$  (05 Marks)



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OR

- 6 a. Find the unit tangent vector and normal vector to the curve  $\vec{r} = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$  at  $x = \frac{1}{\sqrt{2}}$ . (06 Marks)
- b. Find the curl  $(\text{curl } \vec{A})$ , where  $\vec{A} = x^2 y \hat{i} - 2xz \hat{j} + 2yz \hat{k}$  at the point  $(1, 0, 2)$ . (05 Marks)
- c. Show that  $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$  is irrotational. Also find a scalar function of  $\phi$  such that  $\vec{F} = \nabla \phi$ . (05 Marks)

**Module-4**

- 7 a. Obtain the reduction formula for  $\int_0^{\frac{\pi}{2}} \cos^n x \, dx$ . (06 Marks)
- b. Solve  $xy(1+xy^2) \frac{dy}{dx} = 1$ . (05 Marks)
- c. Show that the family of the curves  $y^2 = 4a(x+a)$  is self orthogonal. (05 Marks)

OR

- 8 a. Solve  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$ . (05 Marks)
- b. Evaluate  $\int_0^{\pi} \frac{\sin^4 \theta}{(1 + \cos \theta)^2} d\theta$ . (05 Marks)
- c. If the temperature of the air is  $30^\circ\text{C}$  and a metal ball cools from  $100^\circ\text{C}$  to  $70^\circ\text{C}$  in 15 minutes, find how long will it take for the metal ball to reach a temperature of  $40^\circ\text{C}$ . (06 Marks)

**Module-5**

- 9 a. Find the largest eigen value and the corresponding eigen vector of the matrix  $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ , by using the power method by taking initial vector as  $[1, 1, 1]^T$ . (06 Marks)
- b. Find the rank of the matrix by reducing into the normal form,  $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ . (05 Marks)
- c. Solve the following system of equation by Gauss seidel method:  $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$ . (05 Marks)

OR

- 10 a. Diagonalize the matrix  $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ . (06 Marks)
- b. Solve by Gauss elimination method,  $2x + y + 4z = 12$ ,  $4x + 11y - z = 33$ ,  $8x - 3y + 2z = 20$ . (05 Marks)
- c. Reduce the quadratic form  $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$  into the canonical form. (05 Marks)

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