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First Semester B.E. Degree Examination, Dec.2018/Jan.2019 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find y_n if $y = \frac{1}{x^2 - 5x + 6}$. (06 Marks)
- b. Find the angle between the curves $r = a(1 + \cos \theta)$ $r^2 = a^2 \cos 2\theta$ (05 Marks)
- c. Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a - x)}{x}$ where the curve meets x-axis. (05 Marks)

OR

- 2 a. If $x = \sin t$ $y = \cos t$ prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (n^2 - n^2)y_n = 0$ (06 Marks)
- b. Find the Pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ (05 Marks)
- c. Show that for the curve $r(1 - \cos \theta) = 2a$ ρ^2 varies as r^3 . (05 Marks)

Module-2

- 3 a. Obtain the Taylor's expansion of $\tan^{-1} x$ in powers of $x - 1$ up to the term containing fourth degree. (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right)$. (05 Marks)
- c. If $z = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$ show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$. (05 Marks)

OR

- 4 a. Using Maclaurin's series prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} + \frac{x^4}{24} \dots$ (06 Marks)
- b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (05 Marks)
- c. If $u = \sqrt{x_1 x_2}$ $v = \sqrt{x_2 x_3}$ $w = \sqrt{x_3 x_1}$ find $J \left(\frac{u, v, w}{x_1 x_2 x_3} \right)$. (05 Marks)

Module-3

- 5 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. (05 Marks)
- b. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ (05 Marks)
- c. Show that $\vec{F} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$ is irrotational. Also find a scalar potential such that $\vec{F} = \nabla \phi$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



OR

- 6 a. If $\vec{F} = (3x^2y - z)\mathbf{i} + (xz^3 + y^4)\mathbf{j} - 2x^3z^2\mathbf{k}$ find grad (div \vec{F}) at (2, -1, 0) (06 Marks)
- b. Show that $\vec{F} = \frac{x\mathbf{i} + y\mathbf{j}}{x^2 + y^2}$ is both solenoidal and irrotational. (05 Marks)
- c. Prove curl (grad ϕ) = 0 for any scalar function ϕ . (05 Marks)

Module-4

- 7 a. Obtain reduction formula for $\int_0^{\pi/2} \sin^n x dx$ where n is a positive integer. (06 Marks)
- b. Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x dx$ using reduction formula. (05 Marks)
- c. Solve $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$. (05 Marks)

OR

- 8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x dx$ where n is a positive integer. (06 Marks)
- b. Obtain the orthogonal trajectory of the family of curves $r = a(1 + \sin\theta)$ (05 Marks)
- c. If the temperature of the air is 30°C and metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach temperature of 40°C. (05 Marks)

Module-5

- 9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. (06 Marks)
- b. Solve by Gauss Jordan method $2x + 5y + 7z = 52, 2x + y - z = 0, x + y + z = 9$. (05 Marks)
- c. Find the largest eigen value and the corresponding eigen vector by power method given that $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as $[1 \ 0.8, -0.8]^T$. (05 Marks)

OR

- 10 a. Use Gauss seidel method to solve the equations $x + y + 54z = 110, 27x + 6y - z = 85, 6x + 15y + 2z = 72$. (06 Marks)
- b. Reduce the matrix to diagonal form $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^4 . (05 Marks)
- c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. (05 Marks)
