CBCS Scheme

USN | | | | | | | | | | | 15MAT11

First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x^2}{2x^2+7x+6}$. **(06 Marks)**

b. Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$. **(05 Marks)**

c. Find the radius of curvature of the curve represented by $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(05 Marks)**

OR

2 a. If $y = (x + \sqrt{x^2-1})^m$ then prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of $r^n = a(1 + \cos n\theta)$. **(05 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. **(05 Marks)**

Module-2

3 a. Expand $\sin x$ in powers of $(x - \frac{\pi}{2})$ upto fourth degree term. **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0} \dfrac{xe^x - \log(1+x)}{x^2}$. **(05 Marks)**

c. If $u = x + y + z$, $uv = y + z$, $uvw = z$ then find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **(05 Marks)**

OR

4 a. Find the Maclaurin's series expansion of $\sec x$ upto x^4 term. **(06 Marks)**

b. If $V(x,y) = (1-2xy+y^2)^{-1/2}$ and $x\dfrac{\partial v}{\partial x} - y\dfrac{\partial v}{\partial y} = y^2 V^K$, then find K. **(05 Marks)**

c. If $u = \sin^{-1}\left\{\dfrac{x+2y+3z}{x^8+y^8+z^8}\right\}$ then find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve whose parametric equations are $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the component of its velocity at t = 1 in the direction of I + J + 3K. Find also the component of its acceleration at t = 1 along the normal to I + J + 3K. **(06 Marks)**

b. Verify whether $\vec{A} = (2x + yz)\,I + (4y + zx)\,J - (6z - xy)K$ is irrotational or not. And find the scalar potential of $\vec{A}$. **(05 Marks)**

c. If $\vec{A}$ is a vector point function and ϕ is a scalar point function then prove that $\operatorname{div}(\phi\vec{A}) = \phi \operatorname{div}\vec{A} + (\operatorname{grad}\phi)\cdot\vec{A}$. **(05 Marks)**

OR

6 a. If $\vec{f} = x^2 I + y^2 J + z^2 K$ and $\vec{g} = yzI + zxJ + xyK$, then verify whether $\vec{f} \times \vec{g}$ is solenoidal or not. (06 Marks)

b. Find the directional derivative of $\phi = x^2 + y^2 + 2z^2$ at P(1, 2, 3) in the direction of line $\overrightarrow{PQ} = 4i - 2j + k$. (05 Marks)

c. Prove that curl (grad ϕ) = $\vec{O}$. (05 Marks)

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x \, dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x \, dx$. (06 Marks)

b. Solve $(4xy + 3y^2 - x)\,dx + x(x+2y)dy = 0$. (05 Marks)

c. Find the Orthogonal trajectories of the family $r^n = a^n \sin n\theta$, where a is the parameter. (05 Marks)

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^6 dx}{(4+x^2)^{15/2}}$. (06 Marks)

b. Solve $x\frac{dy}{dx} + y = x^3 y^6$. (05 Marks)

c. A body is heated to 110^0C and placed in air at 10^0C. After one hour its temperature become 60^0C. How much additional time is required for it to cool to 30^0C? (05 Marks)

Module-5

9 a. Solve the following system of equations by Gauss – Jordan method :
$x + y + z = 8$; $-x - y + 2z = -4$; $3x + 5y - 7z = 14$. (06 Marks)

b. Verify the transformation $y_1 = 19x_1 - 9x_2 + 2x_3$; $y_2 = -4x_1 + 2x_2 - x_3$; $y_3 = -2x_1 + x_2$ is regular or not and find the inverse transformation if possible. (05 Marks)

c. Reduce the matrix to the diagonal form
$$A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}.$$ (05 Marks)

OR

10 a. Solve the following system by Gauss – Seidal method : (06 Marks)
$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. Perform three iterations.

b. Determine the largest eigen value and the corresponding eigen vector of
$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$ using Power method. (05 Marks)
Take $(1, 0, 0)^T$ as the initial eigen vector and perform four iterations.

c. Reduce the quadratic form :
$8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. (05 Marks)

* * * * *