USN | | | | | | | | | | 14MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, selecting ONE full question from each module.***

Module-1

1 a. If $y = a\cos(\log_e x) + b\sin(\log_e x)$, show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

b. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cut each other orthogonally. **(06 Marks)**

c. Find the radius of curvature at the point (a, 0) on the curve $xy^2 = a^3 - x^3$. **(07 Marks)**

OR

2 a. Find the n^{th} derivative of $\cos x\cos 2x\cos 3x$. **(06 Marks)**

b. Define curvature of a curve and derive an expression for the radius of curvature in the polar form. **(07 Marks)**

c. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

Module-2

3 a. Obtain the Maclaurin's series for $e^x\cos x$ upto the term containing x^4. **(06 Marks)**

b. If $w = f(x,y)$, $x = r\cos\theta$, $y = r\sin\theta$, show that $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$. **(07 Marks)**

c. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

OR

4 a. Find the constants 'a' and 'b' such that $\lim_{x\to 0}\frac{x(1+a\cos x) - b\sin x}{x^3}$ may be equal to unity. **(07 Marks)**

b. If $u = \log_e\left(\frac{x^5+y^5+z^5}{x^2+y^2+z^2}\right)$, then show that by using Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3$. **(06 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **(07 Marks)**

Module-3

5 a. Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point (1, -1, 2). **(07 Marks)**

b. Show that $\text{Div}(\text{curl}\,\vec{A}) = \vec{0}$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

OR

6 a. A vector field is given by $\vec{f} = (x^2 + xy^2)i + (y^2 + x^2)j$. Show that the field is irrotational and find the scalar potential. **(07 Marks)**

b. If $\vec{r} = xi + yj + zk$, show that i) $\text{div}\,\vec{r} = 3$ ii) $\text{curl}\,\vec{r} = \vec{0}$ **(06 Marks)**

c. Evaluate $\int_0^\infty \left(\frac{e^{-\alpha x}\sin x}{x}\right)dx$ and hence show that $\int_0^\infty \left(\frac{\sin x}{x}\right)dx = \frac{\pi}{2}$, by using differentiation under integral sign. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x\,dx$, where m and n are positive integers. **(06 Marks)**

b. Solve: $xy(1+xy^2)\frac{dy}{dx} = 1$. **(07 Marks)**

c. Find the orthogonal trajectories of a system of confocal and coaxial parabolas $y^2 = 4a(x+a)$. **(07 Marks)**

OR

8 a. Solve: $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. **(07 Marks)**

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}}dx$, using reduction formulae. **(06 Marks)**

c. Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes. **(07 Marks)**

Module-5

9 a. Reduce the following matrix to Echelon form and hence find the Rank,

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(06 Marks)

b. Solve by LU decomposition method $x + 2y + 3z = 14$, $2x + 3y + 4z = 20$, $3x + 4y + z = 14$. **(07 Marks)**

c. Determine the largest eigen-value and the corresponding eigen vector of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ with the initial eigen vector be $[1, 1, 0]^1$ using Rayleigh's power method. Perform six iterations. **(07 Marks)**

OR

10 a. Solve $3x + 8y + 29z = 71$, $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$ by using Gauss-Seidel method. Carryout 3 iterations. **(06 Marks)**

b. Reduce the matrix $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 6xz + 2xy + 2yz$ to the canonical form and specify the matrix of transformation. **(07 Marks)**