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14MAT11

First Semester B.E. Degree Examination, June /July 2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions. selecting ONE full question from each part.***

PART – 1

1 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. (07 Marks)

b. Find the pedal equation of the polar curve $r = a(1 + \cos\theta)$. (06 Marks)

c. Show that the radius of curvature at any point of the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$ is $4a\cos(t/2)$. (07 Marks)

2 a. If $y = \tan^{-1}(x)$ then prove that $(1 + x^2)\, y_{n+2} + (2n + 1)\, xy_{n+1} + n(n+1)\, y_n = 0$. (06 Marks)

b. Find the angle of intersection of curves : $r = \dfrac{a\theta}{1+\theta}$ and $r = \dfrac{\theta}{1+\theta^2}$. (07 Marks)

c. Derive an expression to find radius of curvature in pedal form. (07 Marks)

PART – 2

3 a. Obtain Maclaurin's series for $\log(\sec x)$ upto the term containing x^6. (07 Marks)

b. If u is a homogeneous function of degree 'n' in x and y, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu$. (06 Marks)

c. If $u = f(r, s, t)$ and $r = \dfrac{x}{y}, s = \dfrac{y}{z}, t = \dfrac{z}{x}$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. (07 Marks)

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$. (07 Marks)

b. Find the extreme value of $\sin x + \sin y + \sin(x+y)$. (06 Marks)

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ then find $J\left(\dfrac{x\ y\ z}{r\ \theta\ \phi}\right)$. (07 Marks)

PART – 3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is time. Find the components of velocity and acceleration at $t = 1$ in the direction of $i - 3j + 2k$. (07 Marks)

b. Using differentiation under integral sign rule, evaluate $\int_0^\infty e^{-x^2}\cos(\alpha x)\, dx$. (07 Marks)

c. Apply the general rules to trace a polar curve $r = a(1 + \cos\theta)$. (06 Marks)

6 a. Find the angle between tangent planes $x\log z = y^2 - 1$, $x^2y - 2 - z = 0$ at point (1, 1, 1). (07 Marks)

b. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. (07 Marks)

c. Show that $\operatorname{div}(\operatorname{curl}\vec{F}) = 0$. (06 Marks)

PART – 4

7 a. Obtain the reduction formula for $\int \sin^n x \, dx$. **(07 Marks)**

b. Solve $\sec x \tan x \tan y \, dx + \sec x \sec^2 y \, dy - e^x \, dx = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$. **(07 Marks)**

8 a. Evaluate : $\int_0^{2a} x^3 \sqrt{2ax - x^2} \, dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$. **(06 Marks)**

c. Suppose that an object is heated to 300°F and allowed to cool in a room whose air temperature is 80°F. After 10 minutes the temperature of the object is 250°F. What will be its temperature after 20 minutes? **(07 Marks)**

PART – 5

9 a. Find the rank of matrix :

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}.$$ **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. **(07 Marks)**

c. Use power method to find the largest eigen value and the corresponding eigen vectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ taking initial eigen vectors [1, 1, 1]. **(07 Marks)**

10 a. Solve by Gauss elimination method :

$4x + y + z = 4$
$x + 4y - 2z = 4$
$3x + 2y - 4z = 6$. **(07 Marks)**

b. Show that transformation

$y_1 = 2x_1 + x_2 + x_3$
$y_2 = x_1 + x_2 + 2x_3$
$y_3 = x_1 - 2x_3$ is regular and find the inverse transformation. **(06 Marks)**

c. Solve by LU decomposition method the equations :

$3x + 2y + 7z = 4$
$2x + 3y + z = 5$
$3x + 4y + z = 7$. **(07 Marks)**

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