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First Semester B.E. Degree Examination, Dec.2017/Jan.2018

Engineering Mathematics – I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting ONE full question from each module.**Module – 1**

- 1 a. Find the n^{th} derivative of $\frac{x^2}{(2x+1)(2x+3)}$. (06 Marks)
- b. With the usual notations, prove that $\tan \phi = r \frac{d\theta}{dr}$ and find the angle between the radius vector and the tangent to the curve $r = a(1 - \cos \theta)$ at the point $\theta = \frac{\pi}{3}$. (07 Marks)
- c. Derive an expression to find the radius of curvature in polar form. (07 Marks)
- 2 a. If $y = \cos(m \log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$. (06 Marks)
- b. Find the pedal equation to the curve $\frac{2a}{r} = 1 - \sin \theta$. (07 Marks)
- c. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (07 Marks)

Module – 2

- 3 a. Expand $\log(1+x)$ using Maclaurin's series upto the term containing x^4 . (07 Marks)
- b. State and prove Euler's theorem for homogeneous function of degree n . (06 Marks)
- c. If $u = x + y + z$, $v = y + z$, $z = uvw$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (07 Marks)
- 4 a. (i) Evaluate $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$. (ii) Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$. (06 Marks)
- b. If $u = \sin^{-1} \left(\frac{x^3 + y^3}{x + y} \right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$. (07 Marks)
- c. If $u = f(x - y, y - z, z - x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (07 Marks)

Module – 3

- 5 a. A particle moves along the curve $\vec{r} = \cos 2t \hat{i} + \sin 2t \hat{j} + t \hat{k}$. Find the velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2} \hat{i} + \sqrt{2} \hat{j} + \hat{k}$. (06 Marks)
- b. Show that the vector, $\vec{F} = (3x^2 - 2yz) \hat{i} + (3y^2 - 2xz) \hat{j} + (3z^2 - 2xy) \hat{k}$ is irrotational and find ϕ such that $\vec{F} = \text{grad } \phi$. (07 Marks)
- c. Use general rules to trace the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. (07 Marks)
- 6 a. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (06 Marks)
- b. Show that $\text{curl}(\text{grad } \phi) = 0$. (07 Marks)
- c. By using the rule of differentiation under the integral sign, evaluate $\int_0^{\infty} \frac{e^{-x} \sin(\alpha x)}{x} dx$. (07 Marks)

Module - 4

- 7 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. (06 Marks)
- b. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x dx$. (07 Marks)
- c. Find the orthogonal trajectories of the family of curves $y = x + Ce^{-x}$, where 'C' is the parameter. (07 Marks)
- 8 a. Evaluate $\int_0^1 x^5(1-x^2)^5 dx$. (06 Marks)
- b. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. (07 Marks)
- c. A 12 volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and resistance is 10 ohms. Determine the current if the initial current is zero. (07 Marks)

Module - 5

- 9 a. Solve the following system of equations: $x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$, by Gauss elimination method. (06 Marks)
- b. Diagonalize the matrix $\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$. (07 Marks)
- c. Find the largest eigen value and the corresponding eigen vector of the matrix, $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.
Taking $[1 \ 0 \ 0]^T$ as the initial eigen vector carryout six iterations. (07 Marks)
- 10 a. Solve the following system by LU-decomposition method,
 $x + y + z = 1$, $3x + y - 3z = 5$, $x - 2y - 5z = 10$ (08 Marks)
- b. Find the inverse transformation of,
 $y_1 = 4x_1 + 6x_2 + 6x_3$
 $y_2 = x_1 + 3x_2 + 2x_3$
 $y_3 = -x_1 - 4x_2 - 3x_3$ (06 Marks)
- c. Reduce the quadratic form,
 $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$
to the canonical form. (06 Marks)
