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14MAT11

First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least ONE question from each Part.***

Part – 1

1 a. If $y = e^{ax}\sin(bx+c)$ then prove that $y_n = (a^2+b^2)^{\frac{n}{2}} e^{ax} \sin\left[(bx+c) + n\tan^{-1}\left(\frac{b}{a}\right)\right]$. **(06 Marks)**

b. Show that the radius of curvature at any point of the cycloide $x = a(\theta + \sin\theta)$; $y = a(1-\cos\theta)$ is $4a\cos\left(\frac{\theta}{2}\right)$. **(07 Marks)**

c. Show that the two curves $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$ cut each other orthogonally. **(07 Marks)**

OR

2 a. If $x = \sin t$ and $y = \cos pt$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (p^2-n^2)y_n = 0$. **(07 Marks)**

b. Show that the Pedal equation for the curve $r^m = a^m\cos m\theta$ is $Pa^m = r^{m+1}$ **(06 Marks)**

c. Derive an expression for radius of curvature in polar form. **(07 Marks)**

Part – 2

3 a. If 'u' is a homogenous function of degree 'n' in the variable x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(07 Marks)**

b. Using Maclaurin's series prove that,

$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + ...$$ **(06 Marks)**

c. If z is a function of x and y where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **(07 Marks)**

OR

4 a. If $u = \sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(07 Marks)**

b. Evaluate $\lim_{x\to 0}\left[\frac{a^x+b^x+c^x+d^x}{4}\right]^{\frac{1}{x}}$. **(06 Marks)**

c. If $u = x+y+z$, $uv = y+z$ and $uvw = z$ then show that $\frac{\partial(x\,y\,z)}{\partial(u\,v\,w)} = u^2v$. **(07 Marks)**

Part – 3

5 a. A particle moves along the curve $x = (1-t^3)$, $y = (1+t^2)$, $z = (2t-5)$ determine its velocity and acceleration. Also find the components of velocity and acceleration at t = 1 in the direction of $2i + j + 2k$ **(07 Marks)**

b. Using differentiation under integral sign evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$, $\alpha \ge 0$ **(06 Marks)**

c. Apply the general rules to trace the curve $r = a(1+\cos\theta)$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Apply the general rule to trace curve $y^2(a-x)=x^2(a+x)$, a>0. **(07 Marks)**

b. Show that $\vec{F}=(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational. **(06 Marks)**

c. Show that $\operatorname{div}(\operatorname{curl}A)=0$. **(07 Marks)**

Part – 4

7 a. Obtain the reduction formula for $\int \cos^n x\,dx$ where 'n' being the positive integer. **(07 Marks)**

b. Solve $(y\cos x+\sin y+y)dx+(\sin x+x\cos y+x)dy=0$. **(06 Marks)**

c. Show that the family of curves $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, where λ is a parameter is self orthogonal. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\frac{\pi}{4}} \cos^6 x\sin^6 x\,dx$. **(07 Marks)**

b. Solve $e^y\left(\frac{dy}{dx}+1\right)=e^x$. **(06 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes. The temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Part – 5

9 a. Find the Rank of the matrix $\begin{bmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 8 & 7 & 0 & 5\end{bmatrix}$. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of the given matrix 'A' by using the Rayleigh's power method. Take $[1\ \ 0\ \ 0]^T$ as the initial eigen vector. $A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$ **(06 Marks)**

c. Solve $2x+y+4z=12$, $4x+11y-z=33$ and $8x-3y+2z=20$ by using Gauss Elimination method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method,

$3x+2y+7z=4$

$2x+3y+z=5$

$3x+4y+z=7$

(07 Marks)

b. Reduce the quadratic form $3x^2+5y^2+3z^2-2y^2+2zx-2xy$ the canonical form and specify the matrix of transformation. **(06 Marks)**

c. Show that the transformation $y_1=2x_1+x_2+x_3$, $y_2=x_1+x_2+2x_3$, $y_3=x_1-2x_3$ is regular and also write down the inverse transformation. **(07 Marks)**

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