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10MAT31

Third Semester B.E. Degree Examination, June/July 2019

Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier series expansion of $f(x) = x - x^2$ in $(-\pi, \pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ **(06 Marks)**

b. Find the half range Fourier cosine series of $f(x) = (x-1)^2$ in $0 \le x \le 1$.. **(07 Marks)**

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the following table: **(07 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2 a. Find the Fourier transform of f(x) defined by $f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Find the Fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$. **(07 Marks)**

c. Find the function f(x) whose Fourier cosine transform is given by, $F(\alpha) = \begin{cases} a - \frac{\alpha}{2}, & 0 \le \alpha \le 2a \\ 0, & \alpha > 2a \end{cases}$. **(07 Marks)**

3 a. Obtain the various possible solutions of two dimensional Laplace equation, $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(06 Marks)**

b. Solve the wave equation $u_{tt} = c^2 u_{xx}$ subject to the conditions u(0, t) = 0, u(1, t) = 0, $\frac{\partial u}{\partial t} = 0$ when t = 0 and u(x, 0) = f(x). **(07 Marks)**

c. Obtain the D'Alembert's solution of the one dimensional wave equation $u_{tt} = c^2 u_{xx}$. **(07 Marks)**

4 a. Fit a curve of the form, $y = ab^x$ for the data and hence find the value of y at x = 8.

x	1	2	3	4	5	6	7
y	87	97	113	129	202	195	193

(06 Marks)

b. Solve the following LPP graphically, Maximize $z = 3x + 5y$
Subject to $x + 2y \le 2000$, $x + y \le 1500$, $y \le 600$, $x \ge 0$, $y \ge 0$ **(07 Marks)**

c. Use simplex method to maximize $z = x + 1.5y$
Subject to $x + 2y \le 160$, $3x + 2y \le 240$, $x \ge 0$, $y \ge 0$. **(07 Marks)**

PART – B

5 a. Use Regula Falsi method to find a real root of the equation, $x\log_{10}x - 1.2 = 0$. Carry out three iterations. **(06 Marks)**

b. Use Gauss-Seidal method to solve : $x + 4y + 2z = 15$, $x + 2y + 5z = 20$, $5x + 2y + z = 12$. Perform 3 iterations. **(07 Marks)**

c. Find numerically largest eigen value and the corresponding eigen vector of the matrix, $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as $[1, \ 0.8, \ -0.8]^T$. Perform four iterations. **(07 Marks)**

6 a. A survey conducted in a slum locality reveals the following information as classified below,

Income per day (Rs.)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of persons	20	45	115	210	115

Estimate the probable number of persons in the income group 20 to 25. **(06 Marks)**

b. Use Newton's divided difference formula to find f(15) from the following data: **(07 Marks)**

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

c. Use Simpson's one-third rule to evaluate $\int_2^8 \frac{dx}{\log_{10} x}$ by taking 7 ordinates. **(07 Marks)**

7 a. Solve the wave equation $u_{tt} = 4u_{xx}$ subject to $u(0, t) = 0$, $u(4, t) = 0$, $u_t(x,0) = 0$, $u(x,0) = x(4 - x)$ by taking h = 1, K = 0.5 upto four steps. **(06 Marks)**

b. Find the numerical solution of the parabolic equation $u_{xx} = 2u_t$ when $u(0,t) = 0 = u(4,t)$ and $u(x,0) = x(4 - x)$ by taking h = 1, find the values up to t = 5. **(07 Marks)**

c. Solve Laplace equation $u_{xx} + u_{yy} = 0$ for the following square with boundary values as shown in the following Fig. Q7 (c). **(07 Marks)**

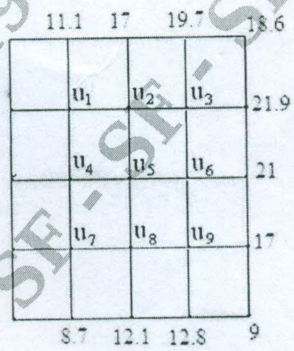

Fig. Q7 (c)

8 a. Find the z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

b. Find the inverse z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(07 Marks)**

c. Solve the difference equation, $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$ using z – transform. **(07 Marks)**

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