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10MAT31

Third Semester B.E. Degree Examination, June/July 2018

Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier Series for the function,

$$f(x)=\begin{cases}\pi x & \text{in } 0\le x\le 1\\ \pi(2-x) & \text{in } 1\le x\le 2\end{cases}.$$ **(07 Marks)**

b. Find the cosine half range series for $f(x)=x(l-x)$; $0\le x\le l$. **(06 Marks)**

c. Obtain the Fourier series of y upto the second harmonics for the following values: **(07 Marks)**

x^0	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	-1.5	0	2.8	3.4

2 a. Find the Fourier transform of $f(x)=e^{-|x|}$. **(07 Marks)**

b. Find the Fourier sine transform of $f(x)=\dfrac{1}{x(1+x^2)}$. **(06 Marks)**

c. Find the Fourier cosine transform of e^{-ax} and deduce that

$$\int_0^\infty \frac{\cos mx}{a^2+x^2}dx=\frac{\pi}{2a}e^{-am}.$$ **(07 Marks)**

3 a. Obtain the various possible solution of one-dimensional wave equation $u_{tt}=C^2u_{xx}$ by the method of separation of variables. **(07 Marks)**

b. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If each of its points is given a velocity $\lambda x(l-x)$. Find the displacement of the string at any distance x from one end at any time t. **(06 Marks)**

c. Solve the Laplace equation, $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=0$

subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\dfrac{n\pi x}{l}$. **(07 Marks)**

4 a. Predict the mean radiation dose at an altitude of 3000 feet by fitting an exponential curve to the given data using $y = ab^x$ **(07 Marks)**

Altitude (x) :	50	450	780	1200	4400	4800	5300
Dose of radiation (y) :	28	30	32	36	51	58	69

b. Using graphical method solve the LPP,

Maximize $z=50x_1+60x_2$,

Subject to the constraints : $2x_1+3x_2\le 1500$,

$3x_1+2x_2\le 1500$,

$0\le x_1\le 400$,

$0\le x_2\le 400$,

$x_1\ge 0,\ x_2\ge 0$. **(06 Marks)**

c. Solve the following minimization problem by simplex method:
Objective function : $P = -3x + 8y - 5z$
Constraints : $-x - 2z \le 5$,
$2x - 3y + z \le 3$,
$2x - 5y + 6z \le 5$,
$x_1, x_2, x_3 \ge 0$. **(07 Marks)**

PART – B

5 a. Using Newton-Raphson iterative formula find the real root of the equation $x \log_{10} x = 1.2$. Correct to five decimal places. **(07 Marks)**

b. Solve, by the relaxation method, the following system of equations:
$9x - 2y + z = 50$
$x + 5y - 3z = 18$
$-2x + 2y + 7z = 19$. **(06 Marks)**

c. Using the Rayleigh's power method find the dominant eigen value and the corresponding eigen vector of the matrix, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ taking $[1,\ 1,\ 1]^T$ as the initial eigen vector. Peform five iterations. **(07 Marks)**

6 a. The population of a town is given by the table. Using Newton's forward and backward interpolation formulae, calculate the increase in the population from the year 1955 to 1985. **(07 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

b. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9, 10 of the independent variable. What is the best estimate you can give for the value of the function at the position 6 of the independent variable? Use Lagrange's method. **(06 Marks)**

c. Use Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule to obtain the approximate value of $\int_0^{0.3} \left(1 - 8x^3\right)^{\frac{1}{2}} dx$ by considering 3 equal intervals. **(07 Marks)**

7 a. Solve numerically the wave equation $u_{xx} = 0.0625u_{tt}$ subject to the conditions, $u(0, t) = 0 = u(5, t)$, $u(x, 0) = x^2(x - 5)$ and $u_t(x, 0) = 0$ by taking $h = 1$ for $0 \le t \le 1$. **(07 Marks)**

b. Solve : $u_{xx} = 32u_t$ subject to the conditions, $u(0, t) = 0$, $u(1, t) = t$ and $u(x, 0) = 0$. Find the values of u upto t = 5 by Schmidt's process taking $h = \frac{1}{4}$. Also extract the following values:
(i) $u(0.75, 4)$ (ii) $u(0.5, 5)$ (iii) $u(0.25, 4)$ **(06 Marks)**

c. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square region shown in the following Fig. Q7 (c), with the boundary values as indicated in the figure. Carry out two iterations. **(07 Marks)**

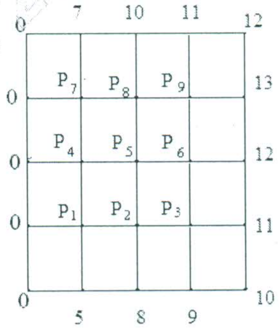

Fig. Q7 (c)

8 a. State initial value property and final value property. If $\bar{u}(z) = \frac{2z^2 + 3z + 4}{(z-3)^3}$, $|z| > 3$. Find the values of u_1, u_2, u_3. **(07 Marks)**

b. Obtain the inverse z-transform of the function,

$$\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}.$$ **(06 Marks)**

c. Solve the difference equation, $y_{n+1} + \frac{1}{4} y_n = \left(\frac{1}{4}\right)^n$, $(n \geq 0)$, $y_0 = 0$ by using z-transform method. **(07 Marks)**

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