10MAT31

Third Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - III

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting atleast TWO questions from each part.*

PART – A

1 a. Obtain Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}.$$

Hence deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dots = \dfrac{\pi^2}{8}$. **(06 Marks)**

b. Obtain Fourier half range Cosine series for the function f(x) = x sin x in (0, π). Hence show

that $\dfrac{1}{1.3} - \dfrac{1}{3.5} + \dfrac{1}{5.7} - \dots\infty = \dfrac{\pi - 2}{4}$. **(07 Marks)**

c. Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier series of f(x) as given in the following table. **(07 Marks)**

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

2 a. Find the Fourier transform of $e^{-a^2x^2}$, a < 0. Hence deduce that $e^{-x^2/2}$ is self reciprocal in respect of Fourier transform. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that

$$\int_0^\infty \frac{x \sin mx}{1 + x^2}\,dx = \frac{\pi e^{-m}}{2}, \ m > 0.$$ **(07 Marks)**

c. Find the Fourier Cosine transform of f(x) = $\dfrac{1}{1 + x^2}$. **(07 Marks)**

3 a. Obtain various possible solutions of the one dimensional Heat equation

$\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(06 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$. Subject to the

conditions u(x, 0) = f(x) and $\dfrac{\partial u}{\partial t}(x,0) = 0$. **(07 Marks)**

c. Obtain various possible solutions of the two dimensional Laplace equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

4 a. Fit a parabola $y = ax^2 + bx + c$ to the following data : **(06 Marks)**

x	0	1	2	3	4	5
y	1	3	7	13	21	31

b. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically. **(07 Marks)**

c. Use Simplex method to solve the following LPP
    Minimize $Z = x_1 - 3x_2 + 3x_3$
    Subject to $3x_1 - x_2 + 2x_3 \leq 7$
    $2x_1 + 4x_2 \geq -12$
    $-4x_1 + 3x_2 + 8x_3 \leq 10$
    $x_1, x_2, x_3 \geq 0.$ **(07 Marks)**

PART – B

5 a. Using Newton – Raphson method, find the value of $\sqrt[3]{18}$ correct to 2 decimals, assuming 2.5 as the initial approximation. **(06 Marks)**

b. Apply Gauss – Seidal iteration method to solve the following equations :
    $3x + 20y - z = -18$; $\quad 2x - 3y + 20z = 25$; $\quad 20x + y - 2z = 17.$ **(07 Marks)**

c. Find the largest Eigen - value and the corresponding Eigen - vector for the matrix
    $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ with initial approximation $[1\ 1\ 0]^T.$ **(07 Marks)**

6 a. Determine $f(x)$ as a polynomial in x for the following data by using Newton's divided difference formula. **(06 Marks)**

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

b. From the data given in the following table, find the number of students who obtained
    i) less than 45 marks and ii) between 40 and 45 marks. **(07 Marks)**

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

c. Evaluate $\int_{4}^{5.2} \log_e x \ dx$ by Weddle's rule. **(07 Marks)**

7 a. Solve the Laplace equation $u_{xx} + u_{yy} = 0$, given that the boundary values for the following square mash. **(06 Marks)**

2 of 3
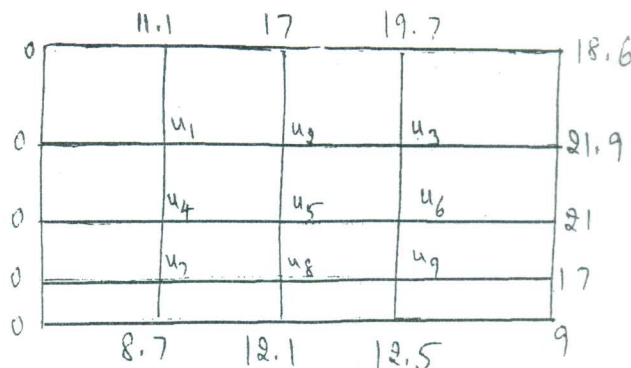
b. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $h = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5, t) = 0$, $u_i(x, 0) = 0$ and $u(x, 0) = x^2(5-x)$. **(07 Marks)**

c. Solve $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ in $0 < x < 5$, $t \geq 0$, given that $u(x, 0) = 20$, $u(0, t) = 0$, $u(5, t) = 100$. Compute u for the time – step with $h = 1$ by Crank – Nicholson method. **(07 Marks)**

8 a. Find the Z – transform of the following :

    i) $(n + 1)^2$ ii) $\sin(3n + 5)$ iii) n_{c_p} $(0 \leq p \leq n)$. **(06 Marks)**

b. If $u(z) = \dfrac{2z^2 + 3z + 12}{(z - 1)^4}$. Find u_0, u_1, u_2, u_3. **(07 Marks)**

c. Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$, $y_1 = 1$, using Z - transforms. **(07 Marks)**

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