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10MAT31

Third Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Find the Fourier series for the function $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$. Hence deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + - - - - -$. **(07 Marks)**

b. Find the half-range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. **(06 Marks)**

c. Obtain the constant term and the co-efficient of the 1st sine and cosine terms in the Fourier series of y as given in the following table. **(07 Marks)**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

2 a. Solve the integral equation :

$$\int_0^\infty f(\theta) \cos \alpha\theta \, d\theta = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}. \text{ Hence evaluate } \int_0^\infty \frac{\sin^2 t}{t^2} dt.$$ **(07 Marks)**

b. Find the Fourier transform of $f(x) = e^{-|x|}$. **(06 Marks)**

c. Find the infinite Fourier cosine transform of e^{-x^2}. **(07 Marks)**

3 a. Solve two dimensional Laplace equation $u_{xx} + u_{yy} = 0$ by the method of separation of variables. **(07 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $u_{tt} = C^2 u_{xx}$ subject to the conditions $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$. **(06 Marks)**

c. Solve the boundary value problem $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < \ell$ subject to the conditions $\frac{\partial u}{\partial x}(0, t) = 0$, $\frac{\partial u}{\partial x}(\ell, t) = 0$, $u(x, 0) = x$. **(07 Marks)**

4 a. Find the equation of the best fit straight line for the following data and hence estimate the value of the dependent variable corresponding to the value of the independent variable x with 30. **(07 Marks)**

x	5	10	15	20	25
y	16	19	23	26	30

b. Solve by graphical method :

Max $Z = x + 1.5y$

Subject to the constraints $x + 2y \le 160$

$3x + 2y \le 240$

$x \ge 0$; $y \ge 0$. **(06 Marks)**

c. Solve by simplex method :

max $z = 3x + 5y$

subject to $3x + 2y \le 18$

$x \le 4$

$y \le 6$

$x, y \ge 0$. **(07 Marks)**

PART – B

5 a. Using the method of false position, find a real root of the equation $x \log_{10} x - 1.2 = 0$, correct to 4 decimal places. **(07 Marks)**

b. By relaxation method, solve :
$10x + 2y + z = 9$; $x + 10y - z = -22$; $-2x + 3y + 10z = 22$. **(06 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector for the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using Rayleigh's power method, taking $x_0 = [1\ 1\ 1]^T$. Perform 5 iterations. **(07 Marks)**

6 a. Find the cubic polynomial by using Newton's forward interpolation formula which takes the following values.

x	0	1	2	3
y	1	2	1	10

Hence evaluate f(4). **(07 Marks)**

b. Using Lagrange's formula, find the interpolating polynomial that approximate the function described by the following table.

x	0	1	2	5
f(x)	2	3	12	147

Hence find f(3). **(06 Marks)**

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Weddler's rule by taking 7 ordinates. **(07 Marks)**

7 a. Solve $u_{xx} + u_{yy} = 0$ in the following square Mesh. Carry out two iterations. **(07 Marks)**

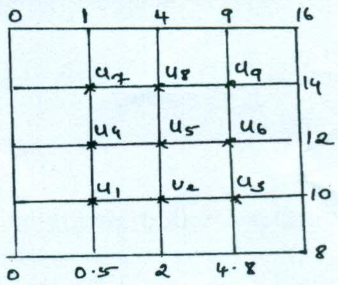

Fig. Q7(a)

b. The transverse displacement of a point at a distance x from one end to any point 't' of a vibrating string satisfies the equation : $\frac{\partial^2 u}{\partial t^2} = 25 \frac{\partial^2 u}{\partial x^2}$ with boundary condition u(0, t) = u(5, t) = 0 and initial condition $u(x, 0) = \begin{cases} 20x & \text{for } 0 \le x \le 1 \\ 5(5-x) & \text{for } 1 \le x \le 5 \end{cases}$ and $u_t(x, 0) = 0$ solve by taking h = 1, k = 0.2 upto t = 1. **(06 Marks)**

c. Find the solution of the equation $u_{xx} = 2u_t$ when u(0, t) = 0 and u(4, t) = 0 and u(x, 0) = x(4 – x) taking h = 1. Find values upto t = 5. **(07 Marks)**

8 a. Find the Z – transformation of the following : i) $3n - 4 \sin \frac{\pi}{4} + 5a^2$ ii) $\frac{a^n e^{-a}}{n!}$. **(07 Marks)**

b. Find the inverse Z – transformation of $\frac{4z^2 - 2z}{z^3 + 5z^2 + 8z - 4}$. **(06 Marks)**

c. Solve the difference equation : $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$; given $y_0 = y_1 = 0$ using Z – transformation. **(07 Marks)**

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