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10MAT31

# Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
## Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

## PART – A

1 a. Obtain the Fourier series in $(-\pi,\pi)$ for $f(x) = x\cos x$. **(07 Marks)**

b. Obtain the Fourier half range sine series,

$$f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}.$$ **(07 Marks)**

c. Obtain the constant term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table. **(06 Marks)**

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 9 | 18 | 24 | 28 | 26 | 20 |

2 a. Find the Fourier transforms of $f(x) = \begin{cases} 1 - x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ and hence evaluate

$$\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx.$$ **(07 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. **(07 Marks)**

c. Find the inverse Fourier sine transform of $\hat{f}_s(\alpha) = \frac{e^{-a\alpha}}{\alpha}$, $a > 0$. **(06 Marks)**

3 a. Solve the wave equation $u_{tt} = c^2 u_{xx}$ given that $u(0,t) = 0 = u(2l,t)$, $u(x, 0) = 0$ and $\frac{\partial u}{\partial t}(x,0) = a\sin^3\frac{\pi x}{2l}$ **(07 Marks)**

b. Solve the boundary value problem $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ $0 < x < l$, $\frac{\partial u}{\partial x}(0,t) = 0$, $\frac{\partial u}{\partial x}(l,t) = 0$, $u(x, 0) = x$. **(07 Marks)**

c. Obtain the D'Almbert's solution of the wave equation, $u_{tt} = C^2 u_{xx}$ subject to the conditions $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(06 Marks)**

4 a. Fit a parabola $y = a + bx + cx^2$ for the data: **(07 Marks)**

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

b. Solve by using graphical method the L.P.P.
Minimize $z = 30x + 20y$
Subject to the constraints: $x - y \leq 1$
$x + y \geq 3$, $y \leq 4$
and $x \geq 0$, $y \geq 0$ **(07 Marks)**

c. Maximize $z = 3x + 4y$
subject to the constraints $2x + y \leq 40$, $2x + 5y \leq 180$,
$x \geq 0, y \geq 0$ using simplex method. **(06 Marks)**

## PART – B

5 a. Find the fourth root of 12 correct to three decimal places by using regula Falsi method. **(07 Marks)**

b. Solve $9x - 2y + z = 50$, $x + 5y - 3z = 18$, $-2x + 2y + 7z = 19$ by relaxation method obtaining the solution correct to two decimal places. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of, $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using power method by taking initial vector as $[1 \quad 1 \quad 1]^T$. **(06 Marks)**

6 a. The table gives the values of $\tan x$ for $0.10 \le x \le 0.30$ **(07 Marks)**

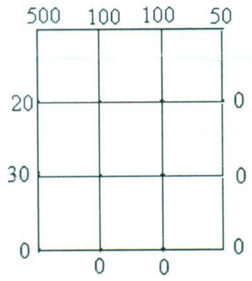
| x | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 |
|---|---|---|---|---|---|
| tanx | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3093 |

b. Using Newton's forward and backward interpolation formula, calculate the increase in population from the year 1955 to 1985. The population in a town is given by, **(07 Marks)**

| Year | 1951 | 1961 | 1971 | 1981 | 1991 |
|---|---|---|---|---|---|
| Population in thousands | 19.96 | 39.65 | 58.81 | 77.21 | 94.61 |

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. Hence deduce the value of $\log_e 2$. **(06 Marks)**

7 a. Solve the Laplace's equation $u_{xx} + u_{yy} = 0$, given that **(07 Marks)**




b. Solve $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0,t) = 0$; $u(4,t) = 0$; $u(x,0) = x(4-x)$. Take h = 1, K = 0.5 upto Four steps. **(07 Marks)**

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x, 0) = \sin \pi x$, $0 \le x \le 1$, $u(0,t) = u(1,t) = 0$ using Schmidt's method. Carry out computations for two levels, taking $h = \frac{1}{3}$, $K = \frac{1}{36}$. **(06 Marks)**

8 a. Find the z-transform of, (i) $\cosh n\theta$ (ii) $\sinh n\theta$ **(07 Marks)**

b. Obtain the inverse z-transform of, $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. **(07 Marks)**

c. Solve the difference equation, $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = y_1 = 0$ using z-transforms. **(06 Marks)**

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