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10MAT31

# Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

## Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

### PART – A

1 a. For the function :

$$f(x)=\begin{cases} x & \text{in} \quad 0<x<\pi \\ x-2\pi & \text{in} \quad \pi<x<2\pi \end{cases}$$

Find the Fourier series expansion and hence deduce the result $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-------$. **(07 Marks)**

b. Obtain the half range Fourier cosine series of the function $f(x)=x(\ell-x)$ in $0\le x\le \ell$. **(06 Marks)**

c. Find the constant term and first harmonic term in the Fourier expansion of y from the following table :

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 9 | 18 | 24 | 28 | 26 | 20 |

**(07 Marks)**

2 a. Find the Fourier transform of the function :

$$f(x)=\begin{cases} 1 & \text{for} \quad |x|\le a \\ 0 & \text{for} \quad |x|>a \end{cases}$$ and hence evaluate : $\int_0^\infty \frac{\sin x}{x}dx$. **(07 Marks)**

b. Obtain the Fourier sine transform of $f(x)=e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}dx,\ m>0$. **(06 Marks)**

c. Solve the integral equation : $\int_0^\infty f(x)\cos px\,dx=\begin{cases} 1-p, & 0\le p\le 1 \\ 0, & p>1 \end{cases}$ and hence deduce the value of $\int_0^\infty \frac{\sin^2 t}{t^2}dt$. **(07 Marks)**

3 a. Obtain the various possible solutions of the two dimensional Laplace's equation $u_{xx}+u_{yy}=0$ by the method of separation of variables. **(07 Marks)**

b. A string is stretched and fastened to two points '$\ell$' apart. Motion is started by displacing the string in the form $y=a\sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time t = 0. Show that the displacement of any point at a distance 'x' from one end at time 't' is given by $y(x,t)=a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi ct}{\ell}\right)$. **(06 Marks)**

c. Obtain the D' Alembert's solution of the wave equation $u_{tt}=c^2u_{xx}$ subject to the conditions $u(x,0)=f(x)$ and $\frac{\partial u}{\partial t}(x,0)=a$. **(07 Marks)**

Important Note : 1. On completing your answers, comp[...] draw diagonal cross lines on the remaining blank p[...]
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will [...]eated as malpractice.

4 a. For the following data fit an exponential curve of the form $y = a e^{bx}$ by the method of least squares :

| x | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| y | 133 | 55 | 23 | 7 | 2 | 2 |

**(07 Marks)**

b. Solve the following LPP graphically :

Minimize $Z = 20x + 10y$

Subject to the constraints : $x + 2y \le 40$

$3x + y \ge 30$

$4x + 3y \ge 60$

$x \ge 0$ and $y \ge 0$. **(06 Marks)**

c. Using Simplex method, solve the following LPP :

Maximize : $Z = 2x + 4y$

Subject to the constraints $3x + y \le 22$

$2x + 3y \le 24$

$x \ge 0$ and $y \ge 0$. **(07 Marks)**

## PART – B

5 a. Using the Regula – Falsi method to find the fourth root of 12 correct to three decimal places. **(07 Marks)**

b. Apply Gauss – Seidal method, to solve the following of equations correct to three decimal places :

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

$27x + 6y - z = 8.5$

(carry out 3 iterations). **(06 Marks)**

c. Using Rayleigh power method, determine the largest eigen value and the corresponding eigen vector, of the matrix A in six iterations. Choose $[1\ 1\ 1]^T$ as the initial eigen vector :

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

**(07 Marks)**

6 a. Using suitable interpolation formulae, find y(38) and y(85) for the following data :

| x | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|
| y | 184 | 204 | 226 | 250 | 276 | 304 |

**(07 Marks)**

b. If $y(0) = -12$, $y(1) = 0$, $y(3) = 6$ and $y(4) = 12$, find the Lagrange's interpolation polynomial and estimate y at x = 2. **(06 Marks)**

c. By applying Weddle's rule, evaluate : $\int_0^1 \frac{x\,dx}{1+x^2}$ by considering seven ordinates. Hence find the value of $\log_e^2$. **(07 Marks)**

7 a. Using finite difference equation, solve $\dfrac{\partial^2 u}{\partial t^2} = 4\dfrac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = u(4, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ upto four time steps. Choose $h = 1$ and $k = 0.5$. **(07 Marks)**

b. Solve the equation $u_t = u_{xx}$ subject to the conditions $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = \sin(\pi x)$ for $0 \le t \le 0.1$ by taking $h = 0.2$. **(06 Marks)**

c. Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown. Find the first iterative values of $u_i (i = 1 - 9)$ to the nearest integer. **(07 Marks)**

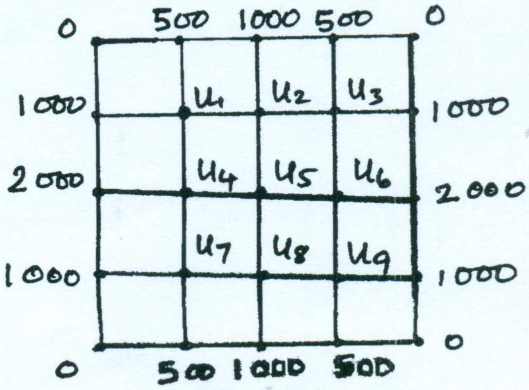

Fig.Q7(c)

8 a. Find the z – transform of $2n + \sin(n\pi/4) + 1$. **(07 Marks)**

b. Obtain the inverse z – transform of $\dfrac{2z^2 + 3z}{(z+2)(z-4)}$. **(06 Marks)**

c. Using z – transform, solve the following difference equation :
$u_{n+2} + 2u_{n+1} + u_n = n$ with $u_0 = u_1 = 0$. **(07 Marks)**

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