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10MAT21

Second Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. Choose correct answers for the following : **(04 Marks)**

i) If $P = \frac{dy}{dx}$, then the differential equation of the first order and higher degree is of the form,

A) $f(y,P)=0$ B) $f(x,P)=0$ C) $f(x,y)=0$ D) $f(x,y,P)=0$

ii) A differential equation of the form, $P^2+P-6=0$ has the general solution,

A) $(y+3x)(y-2x)=0$ B) $(y+3x-c)(y-x-c)=0$

C) $(y+3x-c)(y-2x-c)=0$ D) $(y+x-c)(y-2x-c)=0$

iii) If the given differential equation is solvable for y, takes the form,

A) $y=f(x/P)$ B) $y=f(x,P)$ C) $x=f(y,P)$ D) $y=f(P/x)$

iv) The general solution of the equation, $P=\log_e(Px-y)$ is,

A) $C=\log_e(Cx-y)$ B) $y=\log_e(Cx-y)$

C) $x=\log_e(Cx-y)$ D) None of these

b. Solve $y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ by solving for P. **(04 Marks)**

c. Solve $y-2px=\tan^{-1}(xp^2)$ by solving for y. **(06 Marks)**

d. Solve $x^2(y-px)=yp^2$, use the substitution $X=x^2, Y=y^2$. **(06 Marks)**

2 a. Choose correct answers for the following : **(04 Marks)**

i) A solution of the following differential equation, $(D^2-5D+6)y=0$ is,

A) $y=C_1e^{2x}+C_2e^{3x}$ B) $y=C_1e^{2x}+C_2e^{-3x}$

C) $y=C_1e^{-2x}+C_2e^{3x}$ D) $y=C_1e^{-2x}+C_2e^{-3x}$

ii) Which one of the following solution does not satisfy the differential equation, $\frac{d^3y}{dx^3}-y=0$?

A) e^{-x} B) e^x C) $e^{-\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x$ D) $e^{-\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x$

iii) If m = 2 is a repeated root and m = -1 is another root of the auxiliary equation of a homogeneous differential equation with constant coefficients. The differential equation is,

A) $(D^3+3D^2+4)y=0$ B) $(D^3+3D^2-4)y=0$

C) $(D^3-3D^2+4)y=0$ D) $(D^3-3D^2-4)y=0$

iv) The particular integral for the differential equation, $(D^2+4D+3)y=3e^{2x}$ is,

A) $\frac{1}{15}e^{2x}$ B) $\frac{1}{5}e^{2x}$ C) $3e^{2x}$ D) $5e^{2x}$

b. Solve $(D-2)^2y=8(e^{2x}+\sin 2x)$. **(04 Marks)**

c. Solve $(D^2-1)y=x\sin x+(1+x^2)e^x$. **(06 Marks)**

d. Solve the simultaneous differential equations, $\frac{dx}{dt}=5x+y$, $\frac{dy}{dt}=y-4x$. **(06 Marks)**

3 a. Choose correct answers for the following : (04 Marks)

i) The Wronskian of e^{-x} and xe^{-x} is,

A) e^{-2x} B) $(1-x)e^{-2x}$ C) $-e^{-2x}$ D) $(x-1)e^{-2x}$

ii) The complementary function of the differential equation, $x^2y'' - xy' + y = \log_e x$ is,

A) $\log_e x + 2$ B) $\log x$ C) $x\log_e x + 2$ D) $\frac{x}{2}(\log_e x)^2$

iii) If $D = \frac{d}{dx}$ and $t = \log_e(x+1)$, then the differential equation, $(1+x)^2y'' + (1+x)y' + y = 2\sin\log(1+x)$ becomes,

A) $(D^2+1)y = 2\sin t$ B) $(D^2+1)y = 2\cos t$
C) $(D^2-1)y = 2\sin t$ D) $(D^2-1)y = 2\cos t$

iv) Series solution is a regular singularity of the equation $P_0y'' + P_1y' + P_2y = 0$ when

A) $x > 0$ B) $x < 0$ C) $x = 0$ D) $x \neq 0$

b. Solve $(D^2+1)y = \operatorname{cosec} x$ by the method of variation of parameters. (04 Marks)

c. Solve $(1+x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log_e(1+x)$. (06 Marks)

d. Find the series solution of the differential equation, $\frac{d^2y}{dx^2} + xy = 0$. (06 Marks)

4 a. Choose correct answers for the following : (04 Marks)

i) The order of the partial differential equation by eliminating f from $z = f(x^2+y^2)$ is,

A) Second B) Third C) Zero D) First

ii) The solution of the partial differential equation $x\frac{\partial z}{\partial x} = 2x + y$ is,

A) $z = x^2 + xy + f(y)$ B) $z = 2x + y\log_e x + f(x)$
C) $z = 2x + xy + f(y)$ D) $z = 2x + y\log_e x + f(y)$

iii) The equation of the form $Pp + Qq = R$ is called,

A) Legendre's equation B) Cauchy's equation
C) Euler's equation D) Lagrange's linear equation.

iv) By the method of separation of variables the trial solution of $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ is,

A) $u = X(x)Y(y)$ B) $u = \frac{X(x)}{Y(y)}$ C) $u = \frac{X(x)}{T(t)}$ D) $u = X(x)T(t)$

b. From the partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log_e y\right)$. (04 Marks)

c. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. (06 Marks)

d. Using the method of separation of variables, solve $3\frac{\partial u}{\partial x} + z\frac{\partial u}{\partial y} = 0$, $u(x,0) = 4e^{-x}$ (06 Marks)

PART – B

5 a. Choose correct answers for the following : (04 Marks)

i) The value of $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ is,

A) 0 B) 2 C) π D) $\frac{\pi}{2}$

ii) The double integral $\iint_R r\,dr\,d\theta$ gives,

A) Volume of region R
B) Area of region R
C) Surface area of region R
D) Density of region R.

iii) The value of $\int_0^\infty e^{-x^2}dx$ is,

A) $\sqrt{\pi}$
B) $\frac{\sqrt{\pi}}{2}$
C) π
D) $\frac{\pi}{2}$

iv) The value of $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\,d\theta$ is,

A) $\frac{\pi}{4}$
B) $\frac{\pi\sqrt{2}}{2}$
C) $\frac{\pi}{2}$
D) $\frac{\pi\sqrt{2}}{4}$

b. Evaluate $\int_0^\infty\int_0^x xe^{-x^2/y}dydx$ by changing the order of integration. **(04 Marks)**

c. Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y}\frac{1}{(x+y+z+1)^3}dzdydx$. **(06 Marks)**

d. Show that $\int_0^1\frac{x^2}{\sqrt{1-x^4}}dx.\int_0^1\frac{dx}{\sqrt{1-x^4}}=\frac{\pi}{4}$. **(06 Marks)**

6 a. Choose correct answers for the following : **(04 Marks)**

i) $\int_C \vec{F}\,d\vec{R}$ is the independent of the path joining any two points if and only if it is,

A) Irrotational field
B) Rotational field
C) Solenoidal field
D) None of these

ii) If S is a closed surface enclosing a volume V and if $\vec{R}=x\hat{i}+y\hat{j}+2\hat{k}$, then $\iint_R \vec{R}\,\hat{N}ds$ is,

A) 3V
B) 2V
C) V
D) 4V

iii) The total workdone in moving a particle in a force field $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve $x=t^2+1$, $y=2t^2$, $z=t^3$ from t = 1 to t = 2 is,

A) 18
B) 120
C) 360
D) 303

iv) Stokes theorem connects,

A) a line integral and a surface integral
B) a surface integral and a volume integral
C) a line integral and a volume integral
D) None of these

b. Apply Green's theorem to evaluate $\int_C\left[(xy+y^2)dx+x^2dy\right]$, where C is bounded by $y=x$ and $y=x^2$. **(04 Marks)**

c. Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ taken around the rectangle bounded by the lines, $x=\pm a$, $y=0$, $y=b$. **(06 Marks)**

d. Using divergence theorem evaluate, $\int_S \vec{F}.d\vec{s}$, where $\vec{F}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and s is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant. **(06 Marks)**

7 a. Choose correct answers for the following : **(04 Marks)**

i) If $L[f(t)]=F(s)$, then $L[e^{-at}f(t)]$ is,

A) $-aF(s)$
B) $F(s-a)$
C) $e^{-as}F(s)$
D) $F(s+a)$

ii) The Laplace transform of $\sqrt{t}$ is,

A) $\sqrt{\frac{\pi}{S}}$ B) $\frac{1}{2}\sqrt{\frac{\pi}{S}}$ C) $\frac{\sqrt{\pi}}{2}S^{\frac{3}{2}}$ D) does not exist

iii) $L\left(\int_0^t \cos t dt\right) =$ ____________

A) $\frac{s}{s^2+1}$ B) $\frac{1}{s^2+1}$ C) $\frac{1}{s^2-1}$ D) $\frac{s}{s^2-1}$

iv) The Laplace transform of $\sin 2t\delta\left(t-\frac{\pi}{4}\right)$ is,

A) $e^{\frac{-\pi S}{4}}$ B) $e^{\frac{\pi S}{4}}$ C) $e^{\frac{-\pi S}{2}}$ D) $e^{\frac{\pi S}{2}}$

b. Find the value of $\int_0^\infty te^{-2t}\cos t dt$. (04 Marks)

c. Find the Laplace transform of $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$, where $f(t) = f(t+a)$. (06 Marks)

d. Express f(t) in terms of unit step function and hence find the Laplace transform given that, $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2. \\ t^2, & t > 2 \end{cases}$ (06 Marks)

8 a. Choose correct answers for the following : (04 Marks)

i) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is,

A) Negative integer B) Positive integer C) Negative rational D) None of these

ii) $L^{-1}\left\{\frac{1}{(s+3)^5}\right\} =$ ________

A) $\frac{e^{-3t}t^4}{24}$ B) $\frac{e^{-3t}t^3}{6}$ C) $\frac{e^{-3t}t^2}{2}$ D) $\frac{e^{3t}t^4}{24}$

iii) $L^{-1}\left(\frac{1}{S}\right) =$ __________

A) t B) $\frac{1}{t}$ C) 1 D) $u(t)$

iv) $L^{-1}\left[\frac{se^{-\pi s}}{s^2+a}\right] =$ _________

A) $\cos 3t u(t-\pi)$ B) $-\cos 3t u(t-\pi)$ C) $\frac{1}{3}\cos 3t u(t-\pi)$ D) None of these

b. Find $L^{-1}\left\{\log\frac{s^2+b^2}{s^2+a^2}\right\}$. (04 Marks)

c. Find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$, by using convolution theorem. (06 Marks)

d. Solve by Laplace transform method, given $y'' - 6y' + 9y = t^2e^{3t}$ and $y(0) = 2$, $y'(0) = 6$ (06 Marks)

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