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10MAT21

Second Semester B.E. Degree Examination, June/July 2015
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The general solution for the equation, $x^2p^2+3xyp+2y^2=0$ is,
A) $(x-y-C)(x^2+y^2-C)=0$ B) $(y-x-C)(x-y^2-C)=0$
C) $(xy-C)(x^2y-C)=0$ D) $(y-x-C)(x^2y+C)=0$

ii) The given differential equation is solvable for y, if it is possible to express y in terms of,
A) x and y B) x and p C) y and p D) None of these

iii) The singular solution of the equation, $(y-px)(p-1)=p$ is,
A) $y(1+e^x)+e^x$ B) $x(1+e^x)+e^y$ C) $y(1+e^{-x})+x$ D) $x(1+e^{-x})+e^x+1$

iv) Clairuts equation of $\sin px\cos y=\cos px\sin y+p$ is,
A) $y=px-\sin^{-1}p$ B) $x=py-\cos^{-1}p$ C) $y=xp+\cos^{-1}p$ D) $x=py+\sin^{-1}p$

b. Solve : $p^2+2py\cot x=y^2$. **(04 Marks)**

c. Solve : $x^2+p^2x=yp$. **(06 Marks)**

d. Solve : $y=2px-y^2p^3$. Take $X=2x$, $Y=y^2$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The complimentary function for the differential equation, $y''-6y'+25y=0$ is,
A) $e^{2x}(C_1\cos 3x+C_2\sin 3x)$ B) $e^{-2x}(C_1\cos 3x+C_2\sin 3x)$
C) $e^{3x}(C_1\cos 4x+C_2\sin 4x)$ D) $e^{-3x}(C_1\cos 4x+C_2\sin 4x)$

ii) The displacement in the simple harmonic $\frac{d^2x}{dt^2}=-\mu^2x$ is,
A) $C_1\cos\mu t-C_2\sin\mu t$ B) $C_1\cos\mu t+C_2\sin\mu t$
C) $C_1\cos\mu t\pm C_2\sin\mu t$ D) $\cos\mu t\pm\sin\mu t$

iii) The particular integral of $(D^2+4)y=\cos 2x$ is,
A) $\frac{x\cos 2x}{4}$ B) $\frac{\cos 2x}{8}$ C) $\frac{\sin 2x}{8}$ D) $\frac{x\sin 2x}{4}$

iv) The solution of the differential equation, $y''+3y'+2y=e^{-3x}$ is,
A) $C_1e^{-x}+C_2e^{2x}+\frac{1}{2}e^{-3x}$ B) $C_1e^{x}+C_2e^{-2x}+\frac{1}{2}e^{-3x}$
C) $C_1e^{-x}+C_2e^{-2x}+\frac{1}{2}e^{-3x}$ D) None of these

b. Solve: $\frac{d^2y}{dx}+4y=2^{-x}$. **(04 Marks)**

c. Solve : $\frac{d^3y}{dx^3}+8y=x^2e^{-2x}$ **(06 Marks)**

d. Solve the system: $\frac{dx}{dt}+2x-3y=5t$, $\frac{dy}{dt}-3x+2y=2e^{2t}$. **(06 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) The Wronskian of the differential equation, $(D+2)^2 y = \sec 2x$ is,

A) e^{-2x} B) 2 C) e^{4x} D) e^{-4x}

ii) The complimentary function of the differential equation, $x^2y'' - xy' + y = \log x$ is,

A) $C_1x + C_2x\log x$ B) $C_1x + C_2x^2$ C) $C_1\log x + C_2x^2$ D) $C_1x^2 + C_2x\log x$

iii) The homogeneous linear differential equation whose auxillary equation has roots 1, −1 is,

A) $x^2y_2 - xy_1 + y = 0$ B) $x^2y_2 + xy_1 - y = 0$

C) $x^2y_2 + xy_1 + y = 0$ D) $x^2y_2 - xy_1 - y = 0$

iv) To find the series solution for the equation, $4(1-x)y_2 + 3y_1 + 2y = 0$, we assume the series solution as,

A) $y = \sum_{r=0}^{\infty} a_{r+1}x^{r+1}$ B) $y = \sum_{r=0}^{\infty} a_{R+r}x^{R+r}$ C) $y = \sum_{r=0}^{\infty} a_r x^r$ D) $y = \sum_{r=0}^{\infty} a_r x^{R+r}$

b. By the method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = \frac{1}{1+\sin x}$. (04 Marks)

c. Solve : $(2x+3)^2\frac{d^2y}{dx^2} + 6(2x+3)\frac{dy}{dx} + 6y = \log(2x+3)$. (06 Marks)

d. Obtain the Frobenius-type series solution for the equation, $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$. (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) The partial differential equation obtained from, $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is,

A) $2 = xp + yq$ B) $z = xp + yq$ C) $x = xp + yq$ D) $2z = xp + yq$

ii) The partial differential equation obtained from, $z = e^{my}\phi(x-y)$ is,

A) $px + q = mz$ B) $p + q + mz = 0$ C) $xp + qy = mz$ D) $p + q = mz$

iii) General solution of the equation $\frac{\partial^2 z}{\partial x \partial y} = x^2y$ is,

A) $\frac{x^3y^2}{6} + f(y) + g(x)$ B) $\frac{x^3y^2}{6} + f(y)$

C) $\frac{x^3y^2}{6}$ D) None of these

iv) To solve $u_{xx} - 2u_x + u_t = 0$ by the method of separation of variables, the trial solution is,

A) $X(x)T(x)$ B) $X(x)T(t)$ C) $X(x)\sqrt{T(t)}$ D) $\sqrt{X(x)}T(t)$

b. Form a partial differential equation by eliminating the arbitrary functions f and g from the relation, $z = f(y+2x) + g(y-3x)$. (04 Marks)

c. Solve the equation:

$\frac{\partial^2 z}{\partial x \partial y} + 9x^2y^2 = \cos(2x-y)$ by direct integration, given that z = 0 when y = 0 and $\frac{\partial z}{\partial y} = 0$, (06 Marks)

when x = 0.

d. Solve : $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ (06 Marks)

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_1^4 \int_0^{\sqrt{4-x}} xy\,dy\,dx$ is,

A) $9/2$ B) $3/4$ C) $2/3$ D) $4/5$

ii) $\int_0^1 \int_0^2 \int_1^2 xyz^2\,dx\,dy\,dz =$ ________

A) 2 B) 3 C) 1 D) 3/2

iii) $\int_0^\infty x^3 e^{-4x^2}\,dx =$ ________

A) 21 B) 32 C) 23 D) 1/32

iv) $\Gamma(-7/2) =$ ________

A) $\frac{15}{32}\sqrt{\pi}$ B) $\frac{17}{46}\sqrt{\pi}$ C) $\frac{13}{55}\sqrt{\pi}$ D) $\frac{16}{105}\sqrt{\pi}$

b. Evaluate $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ by changing the order of integration. **(04 Marks)**

c. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\,dy\,dx$ by transforming to polar coordinates. **(06 Marks)**

d. Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\,dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}\,dx = \frac{\pi}{4\sqrt{2}}$ **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\vec{f} = 3xy\,\hat{i} - y^2\,\hat{j}$, then $\int_C \vec{f}\cdot d\vec{r}$ from (0, 0) to (1, 2) along $y = 2x^2$ is,

A) $6/7$ B) $-7/6$ C) $7/6$ D) $-6/7$

ii) If V is the volume obtained by a closed surface S and $\vec{F}$ is a continuously differentiable vector function then $\iiint_V \operatorname{div}\vec{F}\,dv =$ ________

A) 0 B) $\iint_S \vec{F}\times\hat{n}\,ds$ C) $\iint_S \vec{F}\cdot\hat{n}\,ds$ D) None of these

iii) Greens theorem in the plane is $\int_C M\,dx + N\,dy$

A) $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dx\,dy$ B) $\iint_R \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)dx\,dy$ C) $\iint_R \left(\frac{\partial N}{\partial x} + \frac{\partial M}{\partial y}\right)dx\,dy$ D) $\iint_R \left(\frac{\partial N}{\partial y} + \frac{\partial M}{\partial x}\right)dx\,dy$

iv) Stokes theorem is $\int_C \vec{f}.d\vec{r} =$ ________

A) $\int_S (\operatorname{curl}\vec{f})ds$ B) $\int_S (\operatorname{div}\vec{f})ds$ C) $\int_S (\operatorname{curl}\vec{f}).\hat{n}\,ds$ D) None of these

b. Evaluate $\iint_S \vec{f}\cdot\vec{n}\,ds$ where $\vec{f} = yz\,\hat{i} + 2y^2\,\hat{j} + xz^2\,\hat{k}$ and S is the surface of the cylinder $x^2+y^2=9$ contained in the first octant between $z=0$ and $z=2$. **(04 Marks)**

c. Verify Greens theorem for, $\int_C (xy+y^2)dx + x^2dy$, where C is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. **(06 Marks)**

d. Verify Stoke's theorem for $\vec{f} = (2x-y)\hat{i} - yz^2\,\hat{j} - y^2z\,\hat{k}$ for the upper half of the sphere $x^2+y^2+z^2=1$. **(06 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) $L\{t^4 e^{-3t}\} =$ ______

A) $24/s^5$ B) $24/(s-3)^5$ C) $24/(s+3)^5$ D) $24/(s+4)^5$

ii) $L\left\{\frac{1}{\sqrt{t}}\right\} =$ ______

A) $\sqrt{\pi}/s^{3/2}$ B) $-\sqrt{\pi}/s^{1/2}$ C) $\sqrt{\pi}/s^{2/3}$ D) $\sqrt{\pi}/\sqrt{s}$

iii) $\int_0^\infty \frac{e^{-t}\sin t}{t}dt =$ ______

A) $\pi/2$ B) $\pi/4$ C) $\pi/6$ D) $\pi/3$

iv) $L\{e^{(t-1)}H(t-1)\} =$ ______

A) $e^s/s+1$ B) $e^s/s-1$ C) $e^{-s}/s-1$ D) $e^{-s}/s+1$

b. Find the Laplace transform of $\frac{1-\cos t}{t^2}$. (04 Marks)

c. If $f(t) = \begin{cases} t & \text{for } 0 < t \le a \\ 2a - t & \text{for } a < t < 2a \end{cases}$ is a periodic function of period 2a, then prove that $L\{f(t)\} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$. (06 Marks)

d. Express the following function in terms of the unit step function and hence find the Laplace transform: $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$. (06 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) $L^{-1}\left\{\frac{1}{s^{5/2}}\right\} =$ ______

A) $2t^{3/2}/\sqrt{\pi}$ B) $4t^{3/2}/\sqrt{\pi}$ C) $2t^{3/2}/3\sqrt{\pi}$ D) $4t^{3/2}/3\sqrt{\pi}$

ii) $L^{-1}\left\{\frac{s}{s^2-2s+17}\right\} =$ ______

A) $e^t\cos 4t$ B) $\frac{1}{4}e^t\sin 4t$ C) $e^t\cos 4t + \frac{1}{4}e^t\sin 4t$ D) $e^{-t}\cos 4t + \sin 4t$

iii) $L^{-1}\left\{\frac{se^{-\pi s}}{s^2+9}\right\} =$ ______

A) $H(t-\pi)\sin 3t$ B) $H(t-\pi)\cos 3t$ C) $H(t-\pi)\cos 9t$ D) $-H(t-\pi)\cos 3t$

iv) The convolution of two functions f(t) and g(t) is defined by $f(t)*g(t) =$ ______

A) $\int_0^\infty e^{-st}f(u)du$ B) $\int_0^\infty f(u)g(t-u)du$ C) $\int_0^t f(u)g(t-u)du$ D) $\int_0^t e^{-st}g(t-u)du$

b. Find the inverse Laplace transform of $\frac{se^{-s/2}+\pi e^{-s}}{s^2+\pi^2}$. (04 Marks)

c. Using the convolution theorem, obtain the inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$. (06 Marks)

d. Using the Laplace transform method, solve the differential equation, $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 2t^2 + 2t + 2$ under the conditions $y(0) = 2,\ y'(0) = 0$. (06 Marks)

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