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Second Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the answer booklet.*
3. *Answer to objective type questions on sheets other than OMR sheet will not be valued.*

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The general solution of $p^2 - 7p + 12 = 0$ is,
A) $(y+3x-c)(y-4x-c)=0$ B) $(y-3x-c)(y-4x-c)=0$
C) $(y-3x-c)(y+4x-c)=0$ D) None of these

ii) If $y = cx + \frac{a}{c}$ is the general solution of a differential equation then its singular solution is,
A) $x^2 = 4ay$ B) $y = x$ C) $y = -x$ D) $y^2 = 4ax$

iii) The general solution of the differential equation $(p+1)^2(y-px)=1$ is,
A) $y = cx + \frac{1}{(c+1)^2}$ B) $y^2 = 4cx$ C) $x^2 = 4cy$ D) None of these

iv) The general solution of the differential equation $P = \cos y \cos px + \sin y \sin px$ is,
A) $y = c + c\cos^{-1} c$ B) $y = x + \cos^{-1} c$ C) $y = cx + c\cos^{-1} c$ D) $y = cx + c\sin^{-1} c$

b. Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **(05 Marks)**

c. Solve : $y + px = p^2x^4$. **(05 Marks)**

d. Solve : $p^3 - 4xyp + 8y^2 = 0$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The complementary function of $(D-2)^2 y = 8e^{2x}$ is,
A) $y = c_1e^{2x} + c_2e^{-2x}$ B) $y = (c_1 + c_2x)e^{-2x}$
C) $y = c_1e^{-2x} + c_2e^{-2x}$ D) $y = c_1e^{2x} + c_2xe^{2x}$

ii) The particular integral of $(D+1)^2 y = e^{-x}$ is,
A) $\frac{1}{2}x^2e^{-x}$ B) x^2e^{-x} C) $\frac{x^2}{2}e^x$ D) $\frac{x}{2}e^{-x}$

iii) If $f(D) = D^2 - 2$, $\frac{1}{f(D)}e^{2x} =$ __________
A) e^{2x} B) $-\frac{1}{2}e^{2x}$ C) $\frac{1}{2}e^{2x}$ D) $\frac{1}{2}e^{-2x}$

iv) The general solution of $(4D^3 + 4D^2 + D)y = 0$ is,
A) $y = (C_1 + C_2x)e^{\frac{x}{2}} + C_3$ B) $y = (C_1 + C_2x)e^{-\frac{x}{2}} + C_3$
C) $y = (C_1 + C_2x)e^x + C_3$ D) None of these

b. Solve: $(D-2)^2 y = 8(e^{2x} + x^2)$. **(05 Marks)**

c. Solve : $y'' + 4y' + 4y = 3\sin x + 4\cos x$, $y(0) = 1$, $y'(0) = 0$. **(05 Marks)**

d. Solve : $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$. **(06 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) To transform $xy'' + y' = \frac{1}{x}$ into a linear differential equation with constant coefficients, put x = ________

A) e^t B) e^x C) e^{-t} D) e^{-x}

ii) The Wronskian of cosx and sinx is,

A) 0 B) $\frac{\pi}{2}$ C) -1 D) 1

iii) $(x^2D^2 + xD + 7)y = \frac{2}{x}$ converted to a linear differential equation with constant coefficients is,

A) e^t B) $\log(ax+b)$ C) $\frac{d^2y}{dt^2} + 7y = 2e^{-t}$ D) $\frac{d^2y}{dt^2} + 7y = 2e^t$

iv) The solution of $x^2y'' + xy' = 0$ is,

A) $y = C_1 \log x$ B) $y = C_1 \log x + C_2$ C) $y = a\log x + 6$ D) None of these

b. Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by the method of variation of parameters. (05 Marks)

c. Solve : $(1+x)^2 y'' + (1+x)y' + y = 2\sin[\log(1+x)]$. (05 Marks)

d. Solve by Frobenius method the equation, $\frac{d^2y}{dx^2} + y = 0$. (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) The order of the partial differential equation obtained by eliminating f from $z = f(x^2 + y^2)$ is,

A) 0 B) 2 C) 3 D) 1

ii) The partial differential equation obtained from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ by eliminating a and b is,

A) $z = px + qy$ B) $2z = px + qy$ C) $z = px - qy$ D) $2z = px - qy$

iii) The solution of $\frac{\partial^2 z}{\partial x^2} = xy$ is,

A) $z = \frac{x^3y}{6} + xf(y)$ B) $z = \frac{x^3y}{6} + f(y)$

C) $z = \frac{x^3y}{6} + xf(y) + g(y)$ D) None of these

iv) The general solution of $xp + yq = z$ is,

A) $\phi\left(-\frac{x}{y}, \frac{y}{z}\right) = 0$ B) $\phi\left(\frac{x}{y}, -\frac{y}{z}\right) = 0$ C) $\phi(y, z) = 0$ D) $\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

b. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$, when x = 0 and z = 0 if y is an odd multiple of $\frac{\pi}{2}$. (05 Marks)

c. Solve : $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (05 Marks)

d. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x+y)u$. (06 Marks)

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_1^2\int_1^3 xy^2 dxdy$ is,

A) 13 B) 1 C) 0 D) $\frac{23}{2}$

ii) The value of $\int_0^1\int_0^2\int_1^2 x^2yz\ dxdydz$ is,

A) 0 B) 2 C) 1 D) $\frac{1}{2}$

iii) $\int_0^\infty e^{-x^2}dx$ = ____________

A) $\sqrt{\pi}$ B) $\frac{\sqrt{\pi}}{2}$ C) $\frac{2}{\sqrt{\pi}}$ D) $\frac{1}{\sqrt{\pi}}$

iv) $\Gamma(3.5)$ = _________

A) 2.5 B) $\frac{5\sqrt{\pi}}{15}$ C) $\frac{15}{8}$ D) $\frac{15\sqrt{\pi}}{8}$

b. Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)dxdydz$. **(05 Marks)**

c. Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\ dxdy$ and hence evaluate the same. **(05 Marks)**

d. Prove that $\beta\left(m,\frac{1}{2}\right) = 2^{2m-1}\ \beta(m,m)$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) In Green's theorem in the plane $\int_C \phi dx + \psi dy$ = _________

A) $\iint_R\left(\frac{\partial\psi}{\partial x}+\frac{\partial\phi}{\partial y}\right)dxdy$ B) $\iint_R\left(\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}\right)dxdy$ C) $\iint_S \vec{F}.\hat{n}\,ds$ D) $\iint_R\left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right)dxdy$

ii) Relation between line and surface integrals is,

A) Green's theorem B) Stoke's theorem C) Divergence theorem D) None of these

iii) If S is a closed surface enclosing a volume V and if $\vec{R} = xi + yj + zk$, then $\iint_S \vec{R}\cdot N ds$ = __

A) 3V B) 0 C) 1 D) 2V

iv) The area of the ellipse $x = a\cos\theta$, $y = b\sin\theta$ by Green's theorem is,

A) π B) $2\pi ab$ C) πab D) 0

b. Use Green's theorem to evaluate $\int_C[(xy+y^2)dx + x^2dy]$ where C is bounded by y = x and y = x^2. **(05 Marks)**

c. Using Stoke's theorem evaluate, $\int_C[(x+y)dx + (2x-z)dy + (y+z)dz]$ where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) **(05 Marks)**

d. Using divergence theorem evaluate $\int_C \vec{F}\cdot ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4$, z = 0 and z = 3. **(06 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) $L\{t \sin t\} =$ ________

A) $-\frac{2s}{(s^2+1)^2}$ B) $\frac{2s}{(s^2+1)^2}$ C) $\frac{1}{(s^2+1)^2}$ D) $\frac{s}{(s^2+1)^2}$

ii) $L\left\{\frac{1}{\sqrt{t}}\right\} =$ ________

A) $\sqrt{\pi}\sqrt{s}$ B) $\frac{\sqrt{s}}{\sqrt{\pi}}$ C) $2\frac{\sqrt{\pi}}{\sqrt{s}}$ D) $\frac{\sqrt{\pi}}{\sqrt{s}}$

iii) $L\{e^{3(t+1)}\} =$ ________

A) $\frac{e^{-3}}{s-3}$ B) $-\frac{e^3}{s-3}$ C) $\frac{e^3}{s-3}$ D) $\frac{e^3}{s+3}$

iv) $L\{f(t-a)u(t-a)\} =$ ________

A) $e^{-as}f(s)$ B) 0 C) $e^{as}f(s)$ D) $\frac{e^{-as}}{f(s)}$

b. Find $L\{(\sin t - \cos t)^2\}$. (05 Marks)

c. Sketch the graph of the periodic function, $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ and $f(t+2a) = f(t)$, prove that $L\{f(t)\} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$. (05 Marks)

d. Prove that $L\{\delta(t-a)\} = e^{-as}$. (06 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is,

A) 0 B) –ve integer C) +ve integer D) –ve rational

ii) $L^{-1}\left\{\frac{1}{(s+a)^2}\right\} =$ ________

A) e^{at} B) e^{-at} C) t^2e^{at} D) te^{-at}

iii) $L^{-1}\left\{\frac{1}{s^2+4s+13}\right\} =$ ________

A) $e^{-2t}\sin 3t$ B) $e^{-2t}\cos 3t$ C) $\frac{e^{-2t}}{3}\sin 3t$ D) $\frac{e^{-2t}}{3}\cos 3t$

iv) $L^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} =$ ________

A) $-\sin t\, u(t-\pi)$ B) $\cos t\, u(t-\pi)$
C) $\sin(t+\pi)\, u(t-\pi)$ D) $\cos(t-\pi)u(t-\pi)$

b. Find $L^{-1}\left\{\frac{1}{(s+4)^{\frac{5}{2}}} + \frac{1}{\sqrt{2s+3}}\right\}$. (05 Marks)

c. Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$. (05 Marks)

d. Solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x = 2$, $\frac{dx}{dt} = -1$ at $t = 0$. (06 Marks)

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