

10MAT41

(06 Marks)

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018 Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART - A

- 1 a. Use Picards method to obtain the solution of $\frac{dy}{dx} = e^x y$, y(0) = 1 and hence find y(0.2) considering upto third approximation. (06 Marks)
 - b. Using Runge-Kutta method of fourth order find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.2.
 - c. Find y(0.2) using modified Euler's method correct to four decimal places for the equation $\frac{dy}{dx} = x y^2, y(0) = 1, \text{ taking } h = 0.1$ (07 Marks)
- 2 a. Solve $\frac{dy}{dx} = 1 + zx$, $\frac{dz}{dx} = -xy$ with y(0) = 0, z(0) = 1 at x = 0.3 by applying Runge-Kutta method of fourth order.
 - b. Obtain the solution of the equation 2y'' = 4x + y' with initial conditions y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.464, y(1.3) = 2.7514 and y'(1) = 2, y'(1.1) = 2.3178, y'(1.2) = 2.6725 and y'(1.3) = 3.0657 by computing y(1.4) applying Milne's method.
 - c. Use Picard's method to obtain the second approximation to the solution of $\frac{d^2y}{dx^2} x^3 \frac{dy}{dx} x^3y = 0$ given y(0) = 1, $y'(0) = \frac{1}{2}$ and hence find y(0.1). (07 Marks)
- a. State and prove Cauchy-Riemann equations in polar form.
 - Find the analytic function f(z) whose imaginary part is $\left(r \frac{k^2}{r}\right) \sin \theta$, $r \neq 0$ and hence find the real part of f(z).
 - c. If f(z) is a regular function of z, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$. (07 Marks)
- 4 a. Find the image of the triangular region bounded by the lines x = 1, y = 1, x + y = 1 under the transformation $W = Z^2$. (07 Marks)
 - b. If f(z) is analytic within and on C (simple closed curve) and 'a' is a point within 'c' prove that $f(a) = \frac{1}{2\pi i} \int_{C} \frac{f(z)}{z-a} dz$. (06 Marks)
 - c. Evaluate $\int_{C} \frac{e^{2z}}{(z+1)^2(z-2)}$ where C is the circle |z|=3. (07 Marks)



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PART - B

Obtain the series solution of Bessel's differential equation.

(07 Marks)

b. Derive the Rodrigues formula.

(06 Marks)

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$ using Rodrigue's formula find the values of a, b, c, d.

a. If A and B are events with $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{4}$, $P(\overline{B}) = \frac{5}{8}$ find $P(A \cap B)$, $P(\overline{A} \cap \overline{B})$,

 $P(A \cup B)$ and $P(A \cap B)$.

(06 Marks)

- b. In a college boys and girls are equal in proportion. It was found that 10 out of 100 boys and 25 out of 100 girls were referring same author text book. If a student using that was selected at random, what is the probability of being a boy?
- c. A bag contains three coins, one of which is two headed and the other two are normal and fair. A coin is chosen at random from the bag and tossed four times in Succession if head turns up each time, what is the probability that this is the two headed coin.
- a. Find the value of 'K' such that the following distribution represents a finite probability distribution. Hence find the mean (μ) and standard deviation (σ). Also find P(X \le 1), P(X > 1) (06 Marks) and $P(-1 < X \le 2)$.

-3 -2 0 3k 4k 3k 2k k $P(X) \mid k$

- b. If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students are 2.5 and $\sqrt{1.875}$, find an estimate of the number of candidates answering correctly (i) 8 or more questions (ii) 2 or less (iii) 5 questions.
- Derive the expressions for the mean and standard deviation of exponential distribution.

- Certain tubes manufactured by a company have mean life time of 800 hours and standard deviation of 60 hours. Find the probability that a random sample of 16 tubes taken from the group will have mean life time, (i) between 790 hours and 810 hours. (ii) less than 785
 - b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following result.

Horse A: 28 30 32 34 29 | 30 | 30 24 27

Test whether you can discriminate between the two horses. Use $t_{0.05} = 2.2$ and $t_{0.02} = 2.72$

A die is thrown 264 times and the number appearing on the face (x) follows the frequency distribution as mentioned below:

X	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2 .

(07 Marks)