

Sixth Semester B.E. Degree Examination, June/July 2019
Finite Element Methods

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions, selecting
atleast TWO questions from each part.**

PART - A

- 1 a. Derive the 3D equations of equilibrium in elasticity subjected to body force and traction force. (08 Marks)
b. What is FEM? Explain the basic steps involved in FEM. (08 Marks)
c. Explain node numbering scheme and its effect on the half band width. (04 Marks)
- 2 a. Determine the deflection of a cantilever beam of length 'L' and loaded with a vertical load 'P' at the free end by Rayleigh Ritz method use a trial function $Y = a \left(1 - \cos \frac{\pi x}{2h} \right)$. (10 Marks)
b. Use Galerkin's method and obtain an approximate solution of differential equation.

$$\frac{d^2 y}{dx^2} - 10x^2 = 5, \quad 0 \leq x \leq 1$$
with boundary conditions $y(0) = y$ and $y(1) = 0$. (10 Marks)
- 3 a. Derive shape function for 1D bar element in global co-ordinate system. (08 Marks)
b. Derive an expression for Jacobian matrix for a three noded CST element. (08 Marks)
c. Explain 2D - Pascal's triangle. (04 Marks)
- 4 Consider the thin plate shown in Fig.Q4. The plate has a uniform thickness $t = 1$ mm, Young's modulus $E = 200$ GPa and weight density $\rho = 76.6 \times 10^{-6}$ N/mm³. In addition to its weight, the plate is subjected to a point load $P = 100$ N at its mid point.
a. Model the plate with two finite elements.
b. Write down expressions for the elemental stiffness matrices and force vector.
c. Assemble the structural stiffness matrix 'K' and global load vector 'F'.
d. Using elimination approach, solve for the global displacement vector Q.
e. Evaluate the stress in each element.
f. Determine the reaction force at the support. (20 Marks)

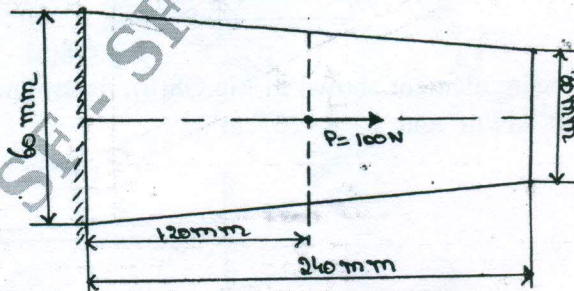


Fig.Q4



PART - B

- 5 a. With a neat sketch explain isoparametric, sub parametric and super parametric elements. (06 Marks)
- b. Write a note on higher order elements used in FEM. (06 Marks)
- c. Using two point Gaussian quadrature formula evaluate the following integral. (08 Marks)

$$I = \int_{-1}^{+1} \int_{-1}^{+1} (r^2 + 2rs + s^2) dr ds$$

- 6 a. List the assumptions made in analysis of truss and also obtain an expression for stiffness matrix of a truss element. (10 Marks)
- b. For the two bar truss shown in Fig.Q6(b), determine the nodal displacements and stress in each member. Take $E = 200 \text{ GPa}$. (10 Marks)

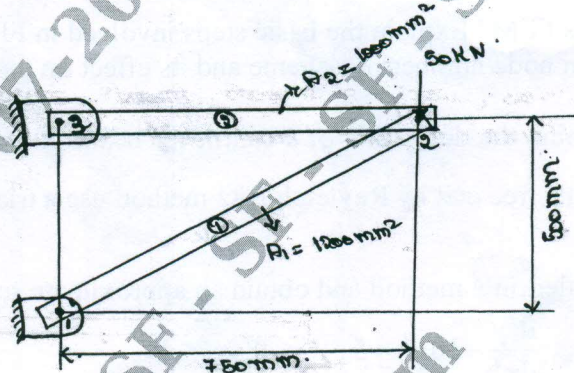
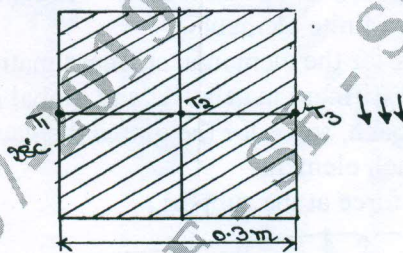


Fig.Q6(b)

- 7 a. Derive elemental stiffness matrix for a beam element in global coordinate system. (10 Marks)
- b. Define Hermite shape function and derive the Hermite shape function for a beam element. (10 Marks)
- 8 a. For the brick wall shown in Fig.Q8(a), the inner surface temperature is 28°C and outer surface is exposed to cold air at -15°C . Determine the temperature distribution in steady state, within the wall by considering two elements, one dimensional heat flow elements. What is heat flux through the wall? (10 Marks)



$k = 0.7 \text{ W/m}^\circ\text{C}$
 $h = 20 \text{ W/m}^2\text{C}$
 $T_{\infty} = -15^\circ\text{C}$

Fig.Q8(a)

- b. For the beam element shown in Fig.Q8(b), determine deflection under the given load. Take $E = 2 \times 10^8 \text{ kN/m}^2$ and $I = 4 \times 10^{-6} \text{ m}^4$. (10 Marks)

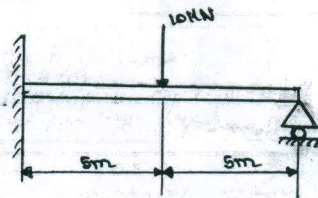


Fig.Q8(b)
