CBCS SCHEME

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18CS36

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r) \to (p \to r)]$ is a tautology. **(06 Marks)**

b. Test the validity of the following argument.

If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.

$\therefore$ I must have watched TV in the evenings **(07 Marks)**

c. Let $p(x) : x^2 - 7x + 10 = 0$, $q(x) : x^2 - 2x + 3 = 0$, $r(x) : x < 0$. Find the truth or falsity of the following statements, when the universe U contains only the integers 2 and 5,

(i) $\forall x, p(x) \to \sim r(x)$ (ii) $\forall x, q(x) \to r(x)$
(iii) $\exists x, q(x) \to r(x)$ (iv) $\exists x, p(x) \to r(x)$ **(07 Marks)**

OR

2 a. Prove that, for any three propositions p, q, r
$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$ **(06 Marks)**

b. Prove that, the following are valid arguments:

(i)
$p \to (q \to r)$
$\sim q \to \sim p$
p

$\therefore r$

(ii)
$\sim p \leftrightarrow q$
$q \to r$
$\sim r$

$\therefore p$

(07 Marks)

c. Give :
(i) a direct proof
(ii) an indirect proof
(iii) proof by contradiction for the following statement.
"If n is an odd integer, then n+9 is an even integer". **(07 Marks)**

Module-2

3 a. Prove that for each $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}n(n+1)(2n+1)$. **(06 Marks)**

b. Determine the coefficient of,
(i) xyz^2 in the expansion of $(2x - y - z)^4$.
(ii) $x^2y^2z^3$ in the expansion of $(3x - 2y - 4z)^7$. **(07 Marks)**

c. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
(i) There is no restriction on the choice.
(ii) Two particular persons will not attend separately.
(iii) Two particular persons will not attend together. **(07 Marks)**

OR

4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and / or 7's. **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four A's are together? How many of them begin with S? **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers, so that, (i) No containers is left empty.
(ii) The fourth container gets an odd number of balls. **(07 Marks)**

Module-3

5 a. For any non empty sets A, B, C prove that,
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $(A \times (B - C)) = (A \times B) - (A \times C)$ **(06 Marks)**

b. Let $f : R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$.

(i) Determine f(0), $f\left(\frac{5}{3}\right)$ (ii) Find $f^{-1}([-5, 5])$. **(07 Marks)**

c. Let f, g, h be functions form z to z defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$. Verify that $(f \circ g) \circ h(x) = f \circ (g \circ h)(x)$. **(07 Marks)**

OR

6 a. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if "a is a multiple of b". Represent the relation R as a matrix and draw its diagraph. **(06 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\}$, define a relation R on $A \times A$, by $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$
(i) Verify that R is an equivalence relation.
(ii) Find the partition of $A \times A$ induced by R. **(07 Marks)**

Module-4

7 a. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. **(06 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(07 Marks)**

c. By using the expansion formula, obtain the rook polynomial for the board C. **(07 Marks)**

		1
	2	3
4	5	6
7	8	

OR

8 a. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have apple. The boy B_3 does not want banana or mango, and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(06 Marks)**

b. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_4 = 37$ satisfy the recurrence relation $a_{n+2} + b a_{n+1} + c a_n = 0$, for $n \geq 0$, find the constants b and c, and solve the relation a_n. **(07 Marks)**

c. How many integers between 1 and 300 (inclusive) are,
(i) Divisible by at least one of 5, 6, 8?
(ii) Divisible by none of 5, 6, 8? **(07 Marks)**

Module-5

9 a. Show that the following two graphs shown in Fig. Q9 (a) – (i) and Fig. Q9 (a) – (ii) are isomorphic, **(06 Marks)**

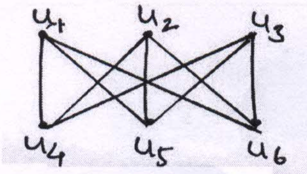

Fig. Q9 (a) – (i)

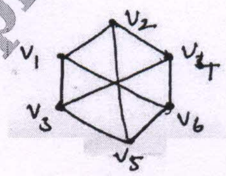

Fig. Q9 (a) – (ii)

b. Define the following with example of each,
(i) Simple graph (ii) Sub graph
(iii) Compliment of a graph (iv) Spanning sub graph **(07 Marks)**

c. Construct an optimal profix code for the symbols a, o, q, u, y, z that occurs with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

OR

10 a. Prove that two simple graphs G_1 and G_2 are isomorphic if and only if their complements are isomorphic. **(06 Marks)**

b. Let G = (V, E) be a simple graph of order $|V| = n$ and size $|E| = m$, if G is a bipartite graph. Prove that $4m \leq n^2$. **(07 Marks)**

c. Construct an optimal prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(07 Marks)**

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