CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define tautology and contradiction. Prove that the compound proposition $[(p \to q) \vee (p \to r) \leftrightarrow [p \to (q \vee r)]$ is tautology. **(05 Marks)**

b. Test the validity of the argument

$p \to q$
$q \to (r \wedge s)$
$\neg r \vee (\neg t \vee u)$
$p \wedge t$
$\therefore u$

(05 Marks)

c. Give (i) direct proof, (ii) indirect proof and (iii) Proof by contradiction, for the statement "Square of an odd integer, is an odd integer". **(06 Marks)**

OR

2 a. Prove the following logical equivalence without using truth table.
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \leftrightarrow \neg(p \vee q)$ **(05 Marks)**

b. Establish the validity of the argument using the rules of inferences.
No engineering student of first or second semester studies Logic
Anil is an engineering student who studies logic.
$\therefore$ Anil is not in second semester. **(05 Marks)**

c. Let $p(x) : x^2 - 7x + 10 = 0$; $q(x) = x^2 - 2x - 3 = 0$; $r(x) = x < 0$
Determine the truth value of the following statements, if universe contains only the integers 2 and 5.
(i) $\forall x, p(x) \to \neg r(x)$ (ii) $\forall x, q(x) \to r(x)$
(iii) $\exists x, p(x) \to r(x)$ (iv) $\exists x, q(x) \to r(x)$ **(06 Marks)**

Module-2

3 a. Prove by mathematical induction for any integer $n \geq 1$.
$$\frac{1}{2.5} + \frac{1}{5.8} + \ldots\ldots\ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$ **(05 Marks)**

b. How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 50,00,000? **(05 Marks)**

c. Find the coefficient of
(i) x^{12} in the expansion of $(1-2x)^{10}\, x^3$
(ii) $x^{11}y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$
(iii) the constant term in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$ **(06 Marks)**

OR

4 a. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. Prove that $a_n \leq 3^n$ for all positive integer n. **(05 Marks)**

b. If $L_0, L_1, L_2, \ldots\ldots$ are Lucas numbers, then prove that

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$ **(05 Marks)**

c. In how many ways can one distribute eight identical bulls into four distinct containers so that (i) no container is left empty? (ii) the fourth container contains an odd number of balls? **(06 Marks)**

Module-3

5 a. Let f, g, h be functions from R to R defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2+2}$, verify that $(h \circ g) \circ f = h \circ (g \circ f)$. **(05 Marks)**

b. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases} \quad \text{then find}$$

(i) $f(-1)$ (ii) $f(5/3)$ (iii) $f^{-1}(3)$ (iv) $f^{-1}(-6)$ (iv) $f^{-1}([-5, 5])$. **(05 Marks)**

c. Define partially ordered set. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

OR

6 a. Determine the following relations are functions or not. If relation is function, find its range

(i) $\{(x, y)/x, y \in z, y = 3x+1\}$; (ii) $\{(x, y)/x, y \in z, y = x^2+3\}$;

(iii) $\{(x, y)/x, y \in R, y^2 = x\}$; (iv) $\{(x, y)/x, y \in Q, x^2+y^2=1\}$ **(05 Marks)**

b. State the Pigeonhole principle and generalization of the pigeonhole principle. Prove that if 30 dictionaries in a library contains a total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages. **(05 Marks)**

c. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as matrix and draw its digraph. **(06 Marks)**

Module-4

7 a. Determine the number of positive integers n, such that $1 \leq n \leq 300$, and n is

(i) not divisible by 5, 6, 8

(ii) divisible by at least one of 5, 6, 8 **(05 Marks)**

b. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party, find that only one chair at each of five tables T_1, T_2, T_3, T_4, T_5 is vacant. P_1 will not sit T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of way they can occupy the vacant chairs. **(05 Marks)**

c. Solve the recurrence relation:

$a_{n+1} = 3a_n + 5 \times 7^{n+1}$ for $n \geq 0$, give that $a_0 = 2$. **(06 Marks)**

OR

8 a. Find the number of permutations of the digits 1 through 9 in which the blocks 36, 78, 672 do not appear. **(06 Marks)**

b. Find the rook polynomial for the board in the Fig.Q8(b). Using expansion formula and product formula. **(06 Marks)**

1	2		
3	4		5
	6	7	8

Fig.Q8(b)

c. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$, satisfy the recurrence relation

$$a_{n+2} + ba_{n+1} + ca_n = 0 \quad \text{for } n \geq 0,$$

determine the constants b and c and then solve the relation for a_n. **(04 Marks)**

Module-5

9 a. Define complete graph and complete bipartite graph. Hence draw
(i) Kuratowaski's first graph K_5,
(ii) Kuratowaski's second graph K_{33}
(iii) 3-regular graph with 8 vertices. **(05 Marks)**

b. Discuss the solution of Kongsberg bridge problem. **(05 Marks)**

c. Obtain an optimal prefix code for the message LETTER RECEIVED. Indicate the code. **(06 Marks)**

OR

10 a. State Handshaking property, how many vertices will a graphs have, if they contain
(i) 16 edges and all vertices of degree 4?
(ii) 21 edges, 3 vertices of degree 4 and other vertices of degree 3?
(iii) 12 edges, 6 vertices of degree 3 and other vertices of degree less than 3. **(05 Marks)**

b. Define isomorphism of two graphs. Show that following pair of graphs are isomorphic. [Refer Fig.Q10(b)].

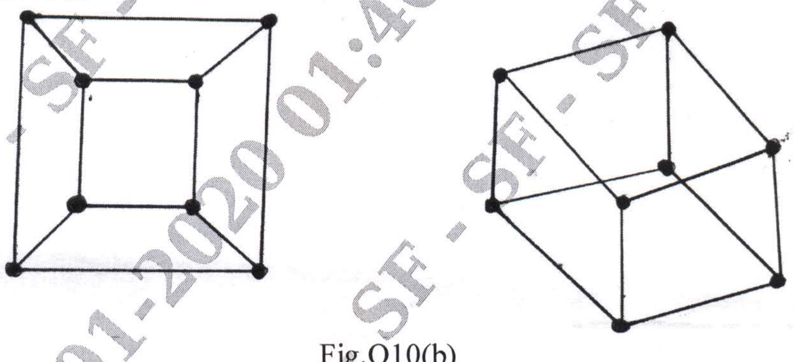

Fig.Q10(b) **(05 Marks)**

c. Define tree and prove that tree with n vertices has n – 1 edges. **(06 Marks)**

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