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18MAT11

First Semester B.E. Degree Examination, June/July 2019 Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. With usual notation, prove that $\tan \phi = r \left(\frac{d\theta}{dr} \right)$. (06 Marks)
- b. Find the radius of curvature of $a^2y = x^3 - a^3$ at the point where the curve cuts the x-axis. (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

- 2 a. Prove that the pedal equation of the curve $r^n = a^n \cos n\theta$ is $a^n \cdot p = r^{n+1}$. (06 Marks)
- b. Show that for the curve $r(1 - \cos\theta) = 2a$, p^2 varies as r^3 . (06 Marks)
- c. Find the angle between the polar curves $r = a(1 - \cos\theta)$ and $r = b(1 + \cos\theta)$. (08 Marks)

Module-2

- 3 a. Expand $\log(1 + \cos x)$ by Maclaurin's series up to the term containing x^4 . (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left\{ \frac{a^x + b^x + c^x}{3} \right\}^{1/x}$. (07 Marks)
- c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

OR

- 4 a. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + z \cdot \frac{\partial u}{\partial z} = 0$. (06 Marks)
- b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at the point $(1, -1, 0)$. (07 Marks)
- c. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box, if the total surface area is minimum. (07 Marks)

Module-3

- 5 a. Evaluate by changing the order of integration $\int_0^a \int_0^{2\sqrt{ax}} x^2 \cdot dy \cdot dx$, $a > 0$. (06 Marks)
- b. Find the area bounded between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$. (07 Marks)
- c. Prove that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$. (07 Marks)



OR

- 6 a. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ (06 Marks)
- b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. (07 Marks)
- c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$ (07 Marks)

Module-4

- 7 a. Solve $(1 + e^{x/y}) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$ (06 Marks)
- b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. (07 Marks)
- c. Solve $yp^2 + (x - y)p - x = 0$. (07 Marks)

OR

- 8 a. Solve $\frac{dy}{dx} + y \cdot \tan x = y^3 \cdot \sec x$ (06 Marks)
- b. Find the orthogonal trajectory of the family of the curves $r^n \cdot \cos n\theta = a^n$, where a is a parameter. (07 Marks)
- c. Solve the equation $(px - y) \cdot (py + x) = 2p$ by reducing into Clairaut's form taking the substitution $X = x^2$, $Y = y^2$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{pmatrix}$ by applying elementary Row transformations. (06 Marks)
- b. Solve the following system of equations by Gauss-Jordan method:
 $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$ (07 Marks)
- c. Using Rayleigh's power method find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ with $X^{(0)} = (1, 0, 0)^T$ as the initial eigen vector carry out 5 iterations. (07 Marks)

OR

- 10 a. For what values of λ and μ the system of equations.
 $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ may have
 i) Unique solution ii) Infinite number of solutions iii) No solution. (06 Marks)
- b. Reduce the matrix $A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$ into diagonal form. (07 Marks)
- c. Solve the following system of equations by Gauss-Seidel method
 $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. Carry out 3 iterations. (07 Marks)
