CBCS SCHEME

USN

17MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x}{(x+1)(2x-3)}$. **(06 Marks)**

b. Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersects orthogonally. **(07 Marks)**

c. Find the Pedal equation of the curve $r = a(1 + \cos\theta)$. **(07 Marks)**

OR

2 a. If $x = \tan y$ prove that $(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. With usual notation, prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(07 Marks)**

c. Find the radius of curvature of the curve $y^2 = \dfrac{a^2(a-x)}{x}$ at the point (a, 0) **(07 Marks)**

Module-2

3 a. Find the Taylor's series of $\log_e x$ about $x = 1$ upto the term containing fourth degree. **(06 Marks)**

b. If $u = \sin^{-1}\left[\dfrac{x^2y^2}{x+y}\right]$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3\tan u$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). **(07 Marks)**

OR

4 a. Evaluate $\displaystyle\lim_{x\to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$. **(06 Marks)**

b. Find the Maclaurin's expansion of $\sqrt{1+\sin 2x}$ upto fourth degree term. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$ **(07 Marks)**

Module-3

5 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ where t denotes time. Find the velocity and acceleration at t = 2. **(06 Marks)**

b. If $\vec{f} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational find a, b, c. Hence find the scalar potential ϕ such that $\vec{f} = \nabla\phi$. **(07 Marks)**

c. Prove that curl (grad ϕ) = 0. **(07 Marks)**

OR

6 a. If $\overrightarrow{f} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ show that $\overrightarrow{f} \cdot \text{curl}\,\overrightarrow{f} = 0$. **(06 Marks)**

b. If $\overrightarrow{f} = \text{grad}(x y^3 z^2)$ find $\text{div}\,\overrightarrow{f}$ and $\text{curl}\,\overrightarrow{f}$. **(07 Marks)**

c. Prove that $\text{div}(\text{curl}\,\overrightarrow{A}) = 0$ **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^a x\sqrt{ax - x^2}\, dx$ **(06 Marks)**

b. Solve $r \sin\theta - \cos\theta \dfrac{dr}{d\theta} = r^2$ **(07 Marks)**

c. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$. **(06 Marks)**

b. Solve $(x^2 + y^2 + x)\, dx + xy\, dy = 0$. **(07 Marks)**

c. Water at temperature 10°C takes 5 minutes to warm upto 20°C in a room temperature 40°C. Find the temperature after 20 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

by reducing it to echelon form. **(06 Marks)**

b. Find the largest eigen value and the corresponding eigen vector for

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

taking $(1\ \ 0\ \ 0)^1$ as initial vector by using power method. (Carry out six iterations) **(07 Marks)**

c. Show that the transformation $y_1 = 2x - 2y - z$, $y_2 = -4x + 5y + 32$ and $y_3 = x - y - z$ is regular and find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the equations $20x + y - 2z = 17$; $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by using Gauss-Seidel method. (Carry out 3 iterations) **(06 Marks)**

b. Diagonalise the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ **(07 Marks)**

c. Reduce the quadratic form $3x^2 - 2y^2 - z^2 + 12yz + 8xz - 4xy$ into canonical form, using orthogonal transformation. **(07 Marks)**

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