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CBCS SCHEME

USN | | | | | | | | | | | **17MAT11**

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics - I

Time: 3 hrs.   Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{x}{(1+x)(1+2x)}$. **(06 Marks)**

b. Prove that the following curves cut orthogonally $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

2 a. If $\cos^{-1}(y/b) = \log(x/n)^n$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$. **(06 Marks)**

b. Find the pedal equation of the curve $r^2 = a^2 \sec 2\theta$. **(07 Marks)**

c. Find the radius of curvature for the curve $y^2 = \dfrac{4a^2(2a-x)}{x}$, where the curve meets the x – axis. **(07 Marks)**

Module-2

3 a. Obtain the Taylor's expansion of $\log_e x$ about $x = 1$ upto the term containing fourth degree. **(06 Marks)**

b. If $u = \operatorname{cosec}^{-1}\left(\dfrac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{6}\tan u$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). **(07 Marks)**

OR

4 a. Evaluate $\underset{x\to 0}{\text{Lt}}\left\{\dfrac{\sin 2x - 2\sin x}{x^3}\right\}$. **(06 Marks)**

b. Obtain the Maclaurin's expansion of the function $\log(1+x)$ upto 4^{th} degree terms. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve, $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. **(06 Marks)**

b. If $\vec{F} = (x+y+az)i + (bx+2y-z)j + (x+cy+2z)k$, find a, b, c such that Curl $\vec{F} = \vec{O}$ and then find ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\operatorname{div}(\phi\vec{A}) = \phi(\operatorname{div}\vec{A}) + \operatorname{grad}\phi.\vec{A}$. **(07 Marks)**

OR

6 a. The position vector of a particle at time t is $\vec{r} = \cos(t-1)\, i + \sin h\,(t-1)\, j + t^3\, k$. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(07 Marks)**

c. Prove that Curl $(\phi\, \vec{A}) = \phi\,(\text{curl}\, \vec{A}) + \text{grad}\, \phi \times \vec{A}$. **(07 Marks)**

Module-4

7 a. Find the reduction formula for $\int_0^{\pi/2} \sin^n x\, dx$. **(06 Marks)**

b. Solve $x^3 \dfrac{dy}{dx} - x^2 y = -y^4 \cos x$. **(07 Marks)**

c. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \dfrac{x^2}{(1+x^2)^{7/2}} dx$. **(06 Marks)**

b. Solve $(y^2 e^{xy^2} + 4x^3)dx + (2xy\, e^{xy^2} - 3y^2)\, dy = 0$. **(07 Marks)**

c. A body in air at 25^0C cools from 100^0C to 75^0C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

(06 Marks)

b. Find the numerically largest eigen value and the corresponding eigen vector of the matrix by power method :

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$ by taking the initial approximation to the eigen vector as [1, 0.8, -0.8]'.

Perform 3 iterations. **(07 Marks)**

c. Show that the transformation :
$y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$ and $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(07 Marks)**

OR

10 a. Solve $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ by Gauss – Seidel method. **(06 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1 x_3$ into Canonical form, using orthogonal transformation. **(07 Marks)**

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