USN | | | | | | | | | |

15MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the nth derivative of $\dfrac{7x + 6}{2x^2 + 7x + 6}$ **(05 Marks)**

   b. Find the angle between the radius vector and the tangent for the curve $r^m = a^m (\cos m\theta + \sin m\theta)$. **(05 Marks)**

   c. Show that the radius of curvature at any point θ on the cycloid $x = a\,(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4\,a\cos(\theta/2)$ **(06 Marks)**

OR

2 a. If $x = \sin t$ and $y = \cos mt$, prove that $(1 - x^2)y_{n+2} - (2n+1)\,xy_{n+1} + (m^2 - n^2)\,y_n = 0$. **(05 Marks)**

   b. Find the pedal equation of the curve $r^2 = a^2 \sec 2\theta$. **(05 Marks)**

   c. Prove with usual notation $\tan\phi = \dfrac{r\,d\theta}{dr}$. **(06 Marks)**

Module-2

3 a. Expand $e^{\sin x}$ using Maclaurin's series upto third degree term. **(05 Marks)**

   b. Evaluate $\displaystyle\lim_{x \to 0}\left[\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right]$. **(05 Marks)**

   c. If $u = e^{(ax+by)}$, $f(ax - by)$, prove that $b\dfrac{\partial u}{\partial x} + a\dfrac{\partial u}{\partial y} = 2abu$ **(06 Marks)**

OR

4 a. Expand $\sin x$ in ascending power of $\pi/2$ upto the term containing x^4. **(05 Marks)**

   b. If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, show that $x\,u_x + y\,u_y = \sin 2u$. **(05 Marks)**

   c. If $u = \dfrac{yz}{x}$, $V = \dfrac{zx}{y}$, $w = \dfrac{xy}{z}$. Find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. **(06 Marks)**

Module-3

5 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$. **(05 Marks)**

   b. Show that $\vec{F} = (y + z)i + (x + z)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(05 Marks)**

   c. Prove that $\nabla \cdot (\phi\vec{A}) = \phi(\nabla \cdot \vec{A}) + \nabla\phi \cdot \vec{A}$. **(06 Marks)**

OR

6 a. Prove that $\text{Curl }(\phi\vec{A}) = \phi(\text{Curl }\vec{A}) + \text{grad}\phi \times \vec{A}$ **(05 Marks)**

   b. A particle moves along the curve C ; $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where 't' denotes the time. Find the component of acceleration at $t = 2$ along the tangent. **(05 Marks)**

c. Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is a conservative force field. Find its scalar potential. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\, dx$. **(05 Marks)**

b. Solve $(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectories of $r = a(1+\sin\theta)$. **(06 Marks)**

OR

8 a. Evaluate $\int_0^2 x\sqrt{2x - x^2}\, dx$ **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. **(05 Marks)**

c. A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F. After half an hour, water cooled to 61°F
 i) What is the temperature of the mineral water in another half an hour?
 ii) How long will it take to cool to 50°F? **(06 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$ **(05 Marks)**

b. Find the largest eigen value and corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ by power method taking $X^{(0)} = [1, 1, 1]^1$ **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(06 Marks)**

10 a. Use Gauss elimination method to solve
 $2x + y + 4z = 12$
 $4x + 11y - z = 33$
 $8x - 3y + 2z = 20$ **(05 Marks)**

b. Find the inverse transformation of the following linear transformation.
 $y_1 = x_1 + 2x_2 + 5x_3$
 $y_2 = 2x_1 + 4x_2 + 11x_3$
 $y_3 = -x_2 + 2x_3$ **(05 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ to the Cannonical form. **(06 Marks)**

* * * * *