CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Find the n^{th} derivative of the $\sin^3 x \cos^2 x$. **(06 Marks)**

b. Find angle between the pair of curves $r = 6\cos\theta$ and $r = 2(1+\cos\theta)$. **(05 Marks)**

c. Show that for the curve $r(1-\cos\theta) = 2a$ the radius of curvature is $\dfrac{2}{\sqrt{a}} r^{3/2}$. **(05 Marks)**

OR

2 a. Show that $\left(\dfrac{2\rho}{a}\right)^2 = \left(\dfrac{x}{y}\right)^2 + \left(\dfrac{y}{x}\right)^2$ for the curve $y = \dfrac{ax}{a+x}$. **(06 Marks)**

b. Find the Pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(05 Marks)**

c. If $y = \log(x + \sqrt{1+x^2})$ prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$. **(05 Marks)**

Module-2

3 a. Expand Log(1+cosx) by Maclaurin's series upto the term containing x^4 **(06 Marks)**

b. Evaluate $\lim\limits_{x \to \pi/2} (\sin x)^{\tan x}$. **(05 Marks)**

c. If $u = \dfrac{yz}{x}$, $v = \dfrac{zx}{y}$, $w = \dfrac{xy}{z}$ show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = u$ **(05 Marks)**

OR

4 a. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$ show that $xu_x + yu_y = \sin 2u$. **(06 Marks)**

b. If $z = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$ show that $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2$ **(05 Marks)**

c. Expand tanx in Taylor's series upto three in powers of $\left(x - \dfrac{\pi}{4}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$, determine velocity and acceleration at $t = 1$. Also find the components of velocity and acceleration in the direction $2i + j + 2k$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$. **(05 Marks)**

c. Prove that $\text{Div}\left(\phi\vec{A}\right) = \phi\left(\text{div}\,\vec{A}\right) + \text{grad}\phi \bullet \vec{A}$ **(05 Marks)**

OR

6 a. Find the unit tangent vector and normal vector to the curve $\vec{r} = \cos 2t\, i + \sin 2t\, j + t\, k$ at $x = \frac{1}{\sqrt{2}}$. **(06 Marks)**

b. Find the $\text{curl}\left(\text{curl}\,\vec{A}\right)$, where $\vec{A} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$ at the point (1, 0, 2). **(05 Marks)**

c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function of ϕ such that $\vec{F} = \nabla\phi$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x\, dx$. **(06 Marks)**

b. Solve $xy(1+xy^2)\frac{dy}{dx} = 1$. **(05 Marks)**

c. Show that the family of the curves $y^2 = 4a(x+a)$ is self orthogonal. **(05 Marks)**

OR

8 a. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(05 Marks)**

b. Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2} d\theta$. **(05 Marks)**

c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(06 Marks)**

Module-5

9 a. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$, by using the power method by taking initial vector as $[1,\ 1,\ 1]^T$ **(06 Marks)**

b. Find the rank of the matrix by reducing into the normal form, $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. **(05 Marks)**

c. Solve the following system of equation by Gauss seidel method: $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$. **(05 Marks)**

OR

10 a. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

b. Solve by Gauss elimination method, $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$. **(05 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into the canonical form. **(05 Marks)**

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