

USN

15MAT11

# First Semester B.E. Degree Examination, June/July 2016 **Engineering Mathematics - I**

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

Find the n<sup>th</sup> derivative of  $y = e^{-3x} \cos^3 x$ .

(06 Marks)

Find the angle of intersection between the curves  $r = a(1 + \sin \theta)$  and  $r = a(1 - \cos \theta)$ .

(05 Marks)

c. Find the radius of curvature at the point  $\left(\frac{3a}{2}, \frac{3a}{2}\right)$  on the curve  $x^3 + y^3 = 3axy$ . (05 Marks)

a. If  $y = \sin(\log(x^2 + 2x + 1))$ , prove that  $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$ .

(06 Marks)

b. Find the pedal equation for the curve  $r^m \cos m\theta = a^m$ .

(05 Marks)

Find the radius of curvature of the curve  $x^4 + y^4 = 2$  at the point (1, 1).

(05 Marks)

a. Expand sin x in powers of  $x - \frac{\pi}{2}$  upto 4<sup>th</sup> degree terms using Taylor's series.

(05 Marks)

b. Evaluate: Limit  $\left(\frac{\tan x}{x}\right)^{1/x^2}$ .

(05 Marks)

c. If  $u = \tan^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$ .

(06 Marks)

a. Expand  $log(1 + e^x)$  using Maclaurin's series upto  $3^{rd}$  degree terms.

(06 Marks)

b. If  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ .

(05 Marks)

c. If  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \theta$ , find  $J\left(\frac{x, y, z}{r \theta \phi}\right)$ .

(05 Marks)

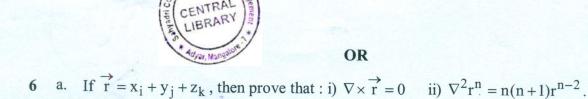
- a. A particle moves along the curve  $x = 2t^2$ ,  $y = t^2 4t$ , z = 3t 5, where t is the time, find the component of its velocity and acceleration in the direction of the vector i - 3j + 2k at t = 1. (06 Marks)
  - Show that  $\overrightarrow{F} = (6xy + z^3)i + (3x^2 z)j + (3xz^2 y)k$  is irrotational, find  $\phi$  such that  $F = \nabla \phi$ .
  - Prove that div(curl u) = 0.

(05 Marks) (05 Marks)



(06 Marks) (05 Marks)

(05 Marks)



b. Prove with usual notations Curl (grad 
$$\phi$$
) = 0

c. Find div  $\overrightarrow{f}$  and curl  $\overrightarrow{f}$  of  $\overrightarrow{f}$  = grad( $x^3 + y^3 + z^3 - 3xyz$ ).

7 a. Obtain the reduction formula of 
$$\int \sin^m x \cos^n x dx$$
. (06 Marks)

b. Solve 
$$(x^2 + y^3 + 6x) dx + y^2x dy = 0$$
. (05 Marks)

c. Find the orthogonal trajectory of 
$$r^n = a^n \cos n\theta$$
, where a is the parameter. (05 Marks)

## OR

8 a. Obtain the reduction formula of 
$$\int \cos^n x \, dx$$
 and hence evaluate:  $\int_0^{\pi/2} \cos^n x \, dx$ . (06 Marks)

b. Solve 
$$\frac{dy}{dx} = xy^3 - xy$$
. (05 Marks)

c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature reaches at 40°C. (Use Newton's law of cooling).

Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$
 (06 Marks)

b. Find the largest eigen value and the corresponding eigen vector of the matrix  $A = \begin{bmatrix} 0 & 2 & 0 \end{bmatrix}$  by power method, use  $\begin{bmatrix} 1, 0 & 0 \end{bmatrix}^T$  as initial vector, take five iterations.

(05 Marks)

c. Reduce the matrix 
$$A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$
 to the diagonal form. (05 Marks)

### OR

10 a. Use Gauss – Siedel iteration method upto 3 iterations to solve with 
$$(0, 0, 0)$$
 as initial values  $10x + y + z = 12$   
 $x + 10y + z = 12$   
 $x + y + 10z = 12$ . (06 Marks)

b. Show that the transformation:

$$y_1 = 2x_1 + x_2 + x_3$$
  
 $y_2 = x_1 + x_2 + 2x_3$   
 $y_3 = x_1 - 2x_3$ 

is regular. Write down the inverse transformation.

c. Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  to the canonical form.

(05 Marks)

(05 Marks)