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CBCS Scheme

USN | | | | | | | | | | 15MAT11

First Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the n^{th} derivative of $y = e^{-3x}\cos^3 x$. **(06 Marks)**

b. Find the angle of intersection between the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \cos\theta)$. **(05 Marks)**

c. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. **(05 Marks)**

OR

2 a. If $y = \sin(\log(x^2 + 2x + 1))$, prove that $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$. **(06 Marks)**

b. Find the pedal equation for the curve $r^m \cos m\theta = a^m$. **(05 Marks)**

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). **(05 Marks)**

Module-2

3 a. Expand sin x in powers of $x - \frac{\pi}{2}$ upto 4^{th} degree terms using Taylor's series. **(05 Marks)**

b. Evaluate: $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$. **(05 Marks)**

c. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. **(06 Marks)**

OR

4 a. Expand $\log(1 + e^x)$ using Maclaurin's series upto 3^{rd} degree terms. **(06 Marks)**

b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time, find the component of its velocity and acceleration in the direction of the vector $i - 3j + 2k$ at $t = 1$. **(06 Marks)**

b. Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational, find ϕ such that $F = \nabla\phi$. **(05 Marks)**

c. Prove that div (curl u) = 0. **(05 Marks)**

OR

6 a. If $\vec{r} = x_i + y_j + z_k$, then prove that : i) $\nabla \times \vec{r} = 0$ ii) $\nabla^2 r^n = n(n+1)r^{n-2}$. **(06 Marks)**

b. Prove with usual notations Curl (grad ϕ) = 0 **(05 Marks)**

c. Find div $\vec{f}$ and curl $\vec{f}$ of $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula of $\int \sin^m x \cos^n x \, dx$. **(06 Marks)**

b. Solve $(x^2 + y^3 + 6x)\,dx + y^2 x\,dy = 0$. **(05 Marks)**

c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$, where a is the parameter. **(05 Marks)**

OR

8 a. Obtain the reduction formula of $\int \cos^n x \, dx$ and hence evaluate : $\int_0^{\pi/2} \cos^n x \, dx$. **(06 Marks)**

b. Solve $\dfrac{dy}{dx} = xy^3 - xy$. **(05 Marks)**

c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature reaches at 40°C. (Use Newton's law of cooling). **(05 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ **(06 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method, use $[1, 0\ 0]^T$. as initial vector, take five iterations. **(05 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. **(05 Marks)**

OR

10 a. Use Gauss – Siedel iteration method upto 3 iterations to solve with (0, 0, 0) as initial values

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12.$$ **(06 Marks)**

b. Show that the transformation :

$y_1 = 2x_1 + x_2 + x_3$

$y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

is regular. Write down the inverse transformation. **(05 Marks)**

c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. **(05 Marks)**

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