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15MAT11

First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1
a. Find the nth derivative of $\dfrac{x^2}{2x^2 + 7x + 6}$. **(06 Marks)**

b. Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$. **(05 Marks)**

c. Find the radius of curvature of the curve represented by $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. **(05 Marks)**

OR

2
a. If $y = (x + \sqrt{x^2 - 1})^m$ then prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of $r^n = a(1 + \cos n\theta)$. **(05 Marks)**

c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. **(05 Marks)**

Module-2

3
a. Expand $\sin x$ in powers of $(x - \dfrac{\pi}{2})$ upto fourth degree term. **(06 Marks)**

b. Evaluate $\lim\limits_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$. **(05 Marks)**

c. If $u = x + y + z$, $uv = y + z$, $uvw = z$ then find $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$. **(05 Marks)**

OR

4
a. Find the Maclaurin's series expansion of $\sec x$ upto x^4 term. **(06 Marks)**

b. If $V(x,y) = (1 - 2xy + y^2)^{-\frac{1}{2}}$ and $x\dfrac{\partial v}{\partial x} - y\dfrac{\partial v}{\partial y} = y^2 V^K$, then find K. **(05 Marks)**

c. If $u = \sin^{-1}\left\{\dfrac{x + 2y + 3z}{x^8 + y^8 + z^8}\right\}$ then find $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$. **(05 Marks)**

Module-3

5
a. A particle moves along the curve whose parametric equations are $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the component of its velocity at $t = 1$ in the direction of $I + J + 3K$. Find also the component of its acceleration at $t = 1$ along the normal to $I + J + 3K$. **(06 Marks)**

b. Verify whether $\vec{A} = (2x + yz) I + (4y + zx) J - (6z - xy)K$ is irrotational or not. And find the scalar potential of $\vec{A}$. **(05 Marks)**

c. If $\vec{A}$ is a vector point function and ϕ is a scalar point function then prove that
$\operatorname{div}(\phi \vec{A}) = \phi \operatorname{div} \vec{A} + (\operatorname{grad}\phi) \cdot \vec{A}$. **(05 Marks)**

OR

6 a. If $\vec{f} = x^2 I + y^2 J + z^2 K$ and $\vec{g} = yzI + zxJ + xyK$, then verify whether $\vec{f} \times \vec{g}$ is solenoidal or not. **(06 Marks)**

   b. Find the directional derivative of $\phi = x^2 + y^2 + 2z^2$ at $P(1, 2, 3)$ in the direction of line $\overrightarrow{PQ} = 4i - 2j + k$. **(05 Marks)**

   c. Prove that curl (grad ϕ) = $\vec{O}$. **(05 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x \, dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x \, dx$. **(06 Marks)**

   b. Solve $(4xy + 3y^2 - x) \, dx + x(x+2y)dy = 0$. **(05 Marks)**

   c. Find the Orthogonal trajectories of the family $r^n = a^n \sin n\theta$, where a is the parameter. **(05 Marks)**

OR

8 a. Evaluate $\int_0^{\infty} \frac{x^6 dx}{(4+x^2)^{15/2}}$. **(06 Marks)**

   b. Solve $x \dfrac{dy}{dx} + y = x^3 y^6$. **(05 Marks)**

   c. A body is heated to $110^0 C$ and placed in air at $10^0 C$. After one hour its temperature become $60^0 C$. How much additional time is required for it to cool to $30^0 C$? **(05 Marks)**

Module-5

9 a. Solve the following system of equations by Gauss – Jordan method :
$x + y + z = 8$; $-x - y + 2z = -4$; $3x + 5y - 7z = 14$. **(06 Marks)**

   b. Verify the transformation $y_1 = 19x_1 - 9x_2 + 2x_3$; $y_2 = -4x_1 + 2x_2 - x_3$; $y_3 = -2x_1 + x_2$ is regular or not and find the inverse transformation if possible. **(05 Marks)**

   c. Reduce the matrix to the diagonal form
$$A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}.$$ **(05 Marks)**

OR

10 a. Solve the following system by Gauss – Seidal method : **(06 Marks)**
$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. Perform three iterations.

   b. Determine the largest eigen value and the corresponding eigen vector of
$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$ using Power method. **(05 Marks)**

Take $(1, 0, 0)^T$ as the initial eigen vector and perform four iterations.

   c. Reduce the quadratic form :
$8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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