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14MAT11

**First Semester B.E. Degree Examination, June/July 2018**  
**Engineering Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, selecting ONE full question from each module.**

**Module-1**

- 1 a. If  $y = a \cos(\log_e x) + b \sin(\log_e x)$ , show that  $x^2 y_2 + xy_1 + y = 0$  and  $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ . (07 Marks)
- b. Show that the curves  $r = a(1 + \cos\theta)$  and  $r = b(1 - \cos\theta)$  cut each other orthogonally. (06 Marks)
- c. Find the radius of curvature at the point  $(a, 0)$  on the curve  $xy^2 = a^3 - x^3$ . (07 Marks)

**OR**

- 2 a. Find the  $n^{\text{th}}$  derivative of  $\cos x \cos 2x \cos 3x$ . (06 Marks)
- b. Define curvature of a curve and derive an expression for the radius of curvature in the polar form. (07 Marks)
- c. Find the pedal equation of the curve  $\frac{2a}{r} = 1 + \cos\theta$ . (07 Marks)

**Module-2**

- 3 a. Obtain the Maclaurin's series for  $e^x \cos x$  upto the term containing  $x^4$ . (06 Marks)
- b. If  $w = f(x, y)$ ,  $x = r \cos\theta$ ,  $y = r \sin\theta$ , show that  $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$ . (07 Marks)
- c. Find the extreme values of the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ . (07 Marks)

**OR**

- 4 a. Find the constants 'a' and 'b' such that  $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3}$  may be equal to unity. (07 Marks)
- b. If  $u = \log_e \left( \frac{x^5 + y^5 + z^5}{x^2 + y^2 + z^2} \right)$ , then show that by using Euler's theorem  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 3$ . (06 Marks)
- c. If  $x = r \sin\theta \cos\phi$ ,  $y = r \sin\theta \sin\phi$ ,  $z = r \cos\theta$ , then find the value of  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . (07 Marks)

**Module-3**

- 5 a. Prove that the surfaces  $4x^2y + z^3 = 4$  and  $5x^2 - 2yz = 9x$  intersect orthogonally at the point  $(1, -1, 2)$ . (07 Marks)
- b. Show that  $\text{Div}(\text{curl } \vec{A}) = \vec{0}$ . (06 Marks)
- c. Use general rules to trace the curve  $y^2(a - x) = x^3$ ,  $a > 0$ . (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.



14MAT11

OR

- 6 a. A vector field is given by  $\vec{f} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2)\mathbf{j}$ . Show that the field is irrotational and find the scalar potential. (07 Marks)
- b. If  $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ , show that i)  $\text{div } \vec{r} = 3$  ii)  $\text{curl } \vec{r} = \vec{0}$  (06 Marks)
- c. Evaluate  $\int_0^{\infty} \left( \frac{e^{-\alpha x} \sin x}{x} \right) dx$  and hence show that  $\int_0^{\infty} \left( \frac{\sin x}{x} \right) dx = \frac{\pi}{2}$ , by using differentiation under integral sign. (07 Marks)

**Module-4**

- 7 a. Obtain the reduction formula for  $\int \sin^m x \cos^n x dx$ , where m and n are positive integers. (06 Marks)
- b. Solve:  $xy(1 + xy^2) \frac{dy}{dx} = 1$ . (07 Marks)
- c. Find the orthogonal trajectories of a system of confocal and coaxial parabolas  $y^2 = 4a(x + a)$ . (07 Marks)

OR

- 8 a. Solve:  $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ . (07 Marks)
- b. Evaluate  $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx$ , using reduction formulae. (06 Marks)
- c. Water at temperature  $100^\circ\text{C}$  cools in 10 minutes to  $88^\circ\text{C}$  in a room of temperature  $25^\circ\text{C}$ . Find the temperature of water after 20 minutes. (07 Marks)

**Module-5**

- 9 a. Reduce the following matrix to Echelon form and hence find the Rank, (06 Marks)
- $$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$
- b. Solve by LU decomposition method  $x + 2y + 3z = 14$ ,  $2x + 3y + 4z = 20$ ,  $3x + 4y + z = 14$ . (07 Marks)
- c. Determine the largest eigen-value and the corresponding eigen vector of  $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  with the initial eigen vector be  $[1, 1, 0]^T$  using Rayleigh's power method. Perform six iterations. (07 Marks)

OR

- 10 a. Solve  $3x + 8y + 29z = 71$ ,  $83x + 11y - 4z = 95$ ,  $7x + 52y + 13z = 104$  by using Gauss-Seidel method. Carryout 3 iterations. (06 Marks)
- b. Reduce the matrix  $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$  to the diagonal form. (07 Marks)
- c. Reduce the quadratic form  $x^2 + 5y^2 + z^2 + 6xz + 2xy + 2yz$  to the canonical form and specify the matrix of transformation. (07 Marks)

\* \* 2 of 2 \* \*