First Semester B.E. Degree Examination, Dec.2015/Jan.2016
Engineering Mathematics – I

Time: 3 hrs.   Max. Marks:100

Note: *Answer FIVE full questions, selecting at least ONE question from each Part.*

Part – 1

1 a. If $y = e^{ax} \sin(bx + c)$ then prove that $y_n = (a^2 + b^2)^{\frac{n}{2}} e^{ax} \sin\left[(bx + c) + n \tan^{-1}\left(\frac{b}{a}\right)\right]$.
   (06 Marks)

   b. Show that the radius of curvature at any point of the cycloide $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$ is $4a\cos\left(\frac{\theta}{2}\right)$. **(07 Marks)**

   c. Show that the two curves $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$ cut each other orthogonally. **(07 Marks)**

OR

2 a. If $x = \sin t$ and $y = \cos pt$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (p^2 - n^2)y_n = 0$. **(07 Marks)**

   b. Show that the Pedal equation for the curve $r^m = a^m \cos m\theta$ is $Pa^m = r^{m+1}$ **(06 Marks)**

   c. Derive an expression for radius of curvature in polar form. **(07 Marks)**

Part – 2

3 a. If 'u' is a homogenous function of degree 'n' in the variable x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(07 Marks)**

   b. Using Maclaurin's series prove that,
   $$\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + \ldots\ldots$$
   (06 Marks)

   c. If z is a function of x and y where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. **(07 Marks)**

OR

4 a. If $u = \sin^{-1}\left[\frac{x^2 + y^2}{x + y}\right]$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(07 Marks)**

   b. Evaluate $\lim\limits_{x \to 0}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{\frac{1}{x}}$. **(06 Marks)**

   c. If $u = x + y + z$, $uv = y + z$ and $uvw = z$ then show that $\frac{\partial(x\,y\,z)}{\partial(u\,v\,w)} = u^2 v$. **(07 Marks)**

Part – 3

5 a. A particle moves along the curve $x = (1 - t^3)$, $y = (1 + t^2)$, $z = (2t - 5)$ determine its velocity and acceleration. Also find the components of velocity and acceleration at t = 1 in the direction of $2i + j + 2k$ **(07 Marks)**

   b. Using differentiation under integral sign evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$, $\alpha \geq 0$ **(06 Marks)**

   c. Apply the general rules to trace the curve $r = a(1 + \cos\theta)$. **(07 Marks)**
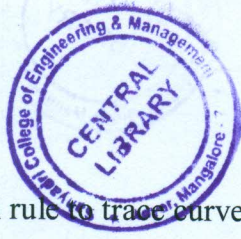
OR

6 a. Apply the general rule to trace curve $y^2(a-x) = x^2(a+x)$, a>0. **(07 Marks)**

b. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational. **(06 Marks)**

c. Show that $div(curlA) = 0$. **(07 Marks)**

Part – 4

7 a. Obtain the reduction formula for $\int \cos^n x\,dx$ where 'n' being the positive integer. **(07 Marks)**

b. Solve $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$. **(06 Marks)**

c. Show that the family of curves $\dfrac{x^2}{a^2 + \lambda} + \dfrac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter is self orthogonal. **(07 Marks)**

OR

8 a. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos^6 x \sin^6 x\,dx$. **(07 Marks)**

b. Solve $e^y\left(\dfrac{dy}{dx} + 1\right) = e^x$. **(06 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes. The temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Part – 5

9 a. Find the Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 0 & 5 \end{bmatrix}$. **(07 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of the given matrix 'A' by using the Rayleigh's power method. Take $[1\ 0\ 0]^T$ as the initial eigen vector. $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **(06 Marks)**

c. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$ and $8x - 3y + 2z = 20$ by using Gauss Elimination method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method,
$3x + 2y + 7z = 4$
$2x + 3y + z = 5$
$3x + 4y + z = 7$ **(07 Marks)**

b. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2y^2 + 2zx - 2xy$ the canonical form and specify the matrix of transformation. **(06 Marks)**

c. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular and also write down the inverse transformation. **(07 Marks)**

* * * * *