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**First Semester B.E. Degree Examination, June/July 2016**  
**Engineering Mathematics – I**

Time: 3 hrs.

Max. Marks:100

- Note: 1. Answer any FIVE full questions, choosing at least two from each part.**  
**2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.**  
**3. Answer to objective type questions on sheets other than OMR will not be valued.**

**PART – A**

- 1 a. Choose the correct answers for the following : (04 Marks)

- i) If  $y = e^{-x^2}$ , then  $y_{n+1} + 2xy_n + 2ny_{n-1}$  is equal to  
 A)  $n^2y$                       B) 0                      C) 2n                      D) 1
- ii) Geometrical meaning of LMVT is that tangent parallel to  
 A) chord                      B) x – axis                      C) y – axis                      D) x = y
- iii) Taylor's series about the origin is  
 A)  $y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \dots$                       B)  $y(0) - xy_1(0) + \frac{x^2}{2}y_2(0) - \dots$   
 C)  $xy_1(0) + \frac{x^2}{2!}y_2(0) + \dots$                       D)  $xy_1(0) - \frac{x^2}{3!} + \frac{x^5}{5!} + y_2(0) + \dots$
- iv) Maclaurin's expansion of  $\cos hx$  is  
 A)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$                       B)  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$                       C)  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$                       D)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

- b. If  $y = e^{a \sin^{-1} x}$ ,  $PT(1-x^2)Y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ . (04 Marks)
- c. State : i) Rolles theorem ii) Lagrange's mean value theorem iii) Cauchy's mean value theorem. (06 Marks)
- d. Using the Maclaurin's series expansion, expand  $\tan x$  up to the term containing  $x^5$ . (06 Marks)

- 2 a. Choose the correct answers for the following : (04 Marks)

- i) Value of  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$  is equal to  
 A)  $\log(a/b)$                       B)  $\log(B/A)$                       C)  $\log(ab)$                       D) 1
- ii) Length of the perpendicular 'p' from the origin to the tangent  
 A)  $r \cos \phi$                       B)  $r \tan \phi$                       C)  $r \sin \phi$                       D)  $r \cot \phi$
- iii) Angle between the radius vector and the tangent is  
 A)  $\tan \phi = \frac{1}{r} \frac{d\theta}{dr}$                       B)  $\cot \phi = r \frac{dr}{d\theta}$                       C)  $\cot \phi = \frac{1}{r} \frac{d\theta}{dr}$                       D)  $\tan \phi = r \frac{d\theta}{dr}$
- iv) Radius of curvature of a circle is  
 A) zero                      B) constant                      C)  $\pi/2$                       D)  $\infty$ .

- b. Evaluate  $\lim_{x \rightarrow 0} \left[ 2 - \frac{x}{a} \right]^{\tan(\frac{\pi x}{2a})}$ . (04 Marks)

- c. Find the angle of intersection of curves  $r = a(1 + \cos \theta)$  and  $r^2 = a^2 \cos 2\theta$ . (06 Marks)

- d. Prove that for the curve  $x = f(x)$  is  $\rho = \frac{[1 + y_1^2]^{3/2}}{y_2}$ . (06 Marks)





3 a. Choose the correct answers for the following :

- i) If  $u = x^y$ , then  $u_{xy}$  is equal to  
 A)  $x^{y-1} (1 + y \log x)$   
 B)  $x^{y-1} y \log x$   
 C)  $y^{y-1} (y + \log x)$   
 D) zero
- ii) If  $A = f_{xx}(a, b)$ ;  $B = f_{xy}(a, b)$ ;  $C = f_{yy}(a, b)$ . Then  $f(x, y)$  will have a maximum at  $(a, b)$   
 A)  $f_x = 0, f_y = 0, AC - B^2 < 0$  and  $A < 0$   
 B)  $f_x = 0, f_y = 0, AC - B^2 < 0$  and  $A > 0$   
 C)  $f_x = 0, f_y = 0, AC - B^2 > 0$  and  $A > 0$   
 D)  $f_x = 0, f_y = 0, AC - B^2 > 0$  and  $A < 0$
- iii) If  $x = r \cos \theta, y = r \sin \theta$ , then  $\frac{\partial(x, y)}{\partial(r, \theta)}$  is equal to  
 A) 1                                      B) r                                      C)  $1/r$                                       D) 0
- iv)  $\delta x = |x - x_0|$  is referred to as  
 A) absolute error in x  
 B) error in x  
 C) relative error in x  
 D) approximate error in x.

b. If  $u = e^{ax-by} \sin(ax + by)$ ; show that  $b \frac{\partial u}{\partial x} - a \frac{\partial u}{\partial y} = 2abu$ . (04 Marks)

c. If  $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ ; show that  $\frac{\partial(uvw)}{\partial(xyz)} = 4$ . (06 Marks)

d. Examine the function  $xy(a - x - y)$  for extreme values. (06 Marks)

4 a. Choose the correct answers for the following :

(04 Marks)

- i) A vector field  $\vec{F}$  is said to irrotational, if  
 A)  $\text{div } \vec{F} = 0$                                       B)  $\text{Curl } \vec{F} = 0$                                       C)  $\text{Grad } \phi = F$                                       D) none
- ii) Given  $\phi = x^2y + y^2z + z^2x$ , then  $\nabla^2 \phi$  is equal to  
 A)  $x + y + z$   
 B)  $2(x + y + z)$   
 C)  $(x^2 + y^2 + z^2)$   
 D)  $2(x - y + z)$
- iii) A vector field  $\vec{F}$  is said to be solenoidal, if  
 A)  $\nabla \phi = F$                                       B)  $\text{div } \vec{F}$                                       C)  $\text{Curl } \vec{F}$                                       D) none
- iv)  $\nabla \times (\nabla \phi)$  is equal to  
 A)  $\nabla^2 \phi$                                       B)  $\nabla \phi$                                       C)  $\vec{0}$                                       D) 0.

b. If A and B are irrotational PT  $A \times B$  is solenoidal. (04 Marks)

c. Show that  $\nabla^2 r^n = n(n+1) r^{n-2}$ . (06 Marks)

d. Prove that  $\text{div } \text{curl } \vec{F} = 0$ . (06 Marks)





**PART - B**

5 a. Choose the correct answers for the following : (04 Marks)

- i) Value of  $\int_{-\pi/2}^{\pi/2} \cos^8 x \, dx$  is equal to  
 A) 0                                      B)  $\frac{35}{64}$                                       C)  $\frac{35}{128}\pi$                                       D)  $\frac{35}{128}$
- ii) If  $f(x,y) = f(-x, -y)$  then the curve is symmetrical about  
 A) origin                                      B) line  $y = x$                                       C) x - axis                                      D) y - axis
- iii) If  $f(r, \theta) = f(r, \pi - \theta)$  then the curve is symmetrical about  
 A) line  $\theta = \frac{\pi}{4}$                                       B)  $\theta = \frac{\pi}{2}$                                       C)  $\theta = 0$                                       D)  $\theta = \frac{\pi}{3}$
- iv) Parametric equation for  $x^{2/3} + y^{2/3} = a^{2/3}$  (Astroid) is  
 A)  $x = a \cos^3 \theta, y = a \sin^3 \theta$                                       B)  $x = a \cos^2 \theta, y = a \sin^2 \theta$   
 C)  $x = \cos^3 \theta, y = \sin^3 \theta$                                       D) none.

b. Given :  $\int_0^\pi \frac{dx}{a + b \cos x} = \frac{\pi}{\sqrt{a^2 + b^2}}$  ( $a > b$ ); evaluate  $\int_0^\pi \frac{dx}{(a + b \cos \theta)^2}$ . (04 Marks)

c. Obtain the reduction formula for  $\int_0^{\pi/2} \sin^n \theta \, d\theta$ . (06 Marks)

d. Find the area of the cardioid  $r = a(1 - \cos \theta)$  (06 Marks)

6 a. Choose the correct answers for the following : (04 Marks)

- i) Substitution that transformations the equation :  $\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y + 3}$  to homogeneous form is  
 A)  $x - y = t$                                       B)  $(y/x) = t$                                       C)  $x/y = t$                                       D)  $x + y = t$
- ii) Integrating factor for the D.E.  $\frac{dy}{dx} + Py = Q$ , where P and Q are function x only  
 A)  $e^{\int P \, dy}$                                       B)  $\int P \, dx$                                       C)  $\int P \, dy$                                       D)  $e^{\int P \, dx}$
- iii) Necessary and sufficient condition for the DE  $M(x, y) \, dx + N(x, y) \, dy = 0$  to be an exact is  
 A)  $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$                                       B)  $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial y} = 0$                                       C)  $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$                                       D) none
- iv) The family of straight lines passing through the origin is represented by the differential equation :  
 A)  $y \, dx + x \, dy = 0$                                       B)  $x \, dx + y \, dy = 0$                                       C)  $x \, dy - y \, dx = 0$                                       D) none.

b. Solve  $\left(1 + e^{x/y}\right) dx + e^{x/y} \left(1 - \frac{x}{y}\right) dy = 0$ . (04 Marks)

c. Solve  $(1 + y^2) dx = (\tan^{-1} y - x) \, dy$ . (06 Marks)

d. ST the family of parabolas  $y^2 = 4a(x + a)$  is self orthogonal. (06 Marks)





7 a. Choose the correct answers for the following :

(04 Marks)

i) Rank of  $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$  is

- A) 2                                      B) 1                                      C) -3                                      D) 4
- ii) A set of 'm' linear equations, with n unknowns posses infinite solution if
  - A)  $\rho(A) = \rho[A : B] = r = n$                                       B)  $\rho(A) = \rho[A : B] = r < n$
  - C)  $\rho(A) \neq \rho[A : B]$                                       D)  $\rho(A) = \rho[A : B] = r > n$
- iii) For a system of linear homogeneous equation if  $\rho(A) = \rho[A : B] = n$ , where n is the number of un known system was
  - A) trivial solution              B) non trivial solution C) both A and B              D) no solution
- iv) For non homogeneous system of linear equations, Gauss lamination method is applicable, of the coefficient matrix is reduced to
  - A) Symmetric matrix                                      B) lower triangular matrix
  - C) diagonal matrix                                      D) upper triangular matrix.

b. Using the elementary transformation reduce the matrix A to Echelon form  $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ .

Find the rank of the matrix A.

(04 Marks)

- c. Investigate the values of  $\lambda$  and  $\mu$  so that the equations :  $x + y + z = 6$ ,  $x + 2y + 3z = 10$ ,  $x + 2y + \lambda z = \mu$ , have i) no solution ii) unique solution iii) infinite number of solutions. (06 Marks)
- d. Solve the system of equations by Gauss Jordan method :  $x + y + z = 9$ ,  $x - 2y + 3z = 8$   $2x + y - z = 3$ . (06 Marks)

8 a. Choose the correct answers for the following :

(04 Marks)

- i) Linear transformation  $Y = AX$  is regular, if
  - A) A is singular              B) A is square              C) A is non singular              D) none
- ii) Sum of eigen values of a square matrix is equal to
  - A) Sum of the principle diagonal elements
  - B) product of principle diagonal elements
  - C) determine of value of that matrix
  - ~~D) none~~
- iii) The matrix B of same order as A is said to be similar if these exist D such that
  - A)  $A = P^{-1}BP$               B)  $B = P^{-1}AP$               C)  $A^n = PD^n P^{-1}$               D)  $B = P^{-1}DP$
- iv) Matrix 'D' which diagonalises 'A' is
  - A) Spectral matrix of A                                      B) Orthogonal matrix of A
  - C) null matrix                                      D) modal matrix of A.

b. Show that the transformation,  $y_1 = 2x_1$   $x_2 + x_3$ ,  $y_2 = x_1 + x_2 + 2x_3$ ,  $y_3 = x_1 - 2x_3$ , is Regular and write down the inverse transformation. (04 Marks)

c. Reduce the matrix  $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$  to the diagonal form. (06 Marks)

d. Find the rank, index, signature of the following quadratic form :  $2x^2 - 2y^2 + 2z^2 - 2xy - 8yz + 6zx$ . (06 Marks)

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