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14MAT11

First Semester B.E. Degree Examination, June/July 2015

Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

MODULE– I

1 a. If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ (07 Marks)

b. Find the pedal equation for the curve $r^m = a^m \sin m\theta + b^m \cos m\theta$ (06 Marks)

c. Derive an expression to find radius of curvature in cartesian form. (07 Marks)

OR

2 a. Find the n^{th} derivative of $\sin^2 x \cos^3 x$ (07 Marks)

b. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect at right angles. (06 Marks)

c. Find the radius of curvature when $x = a \log(\sec t + \tan t)$, $y = a \sec t$. (07 Marks)

MODULE– II

3 a. Using Maclaurin's series expand $\tan x$ upto the term containing x^5. (07 Marks)

b. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2u \log u$ where $\log u = \dfrac{x^3 + y^3}{3x + 4y}$ (06 Marks)

c. Find the extreme values of $x^4 + y^4 - 2(x - y)^2$ (07 Marks)

OR

4 a. Evaluate $\lim_{x\to 0} \left\{\dfrac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}\right\}$ (07 Marks)

b. If $u = x \log xy$ where $x^3 + y^3 + 3xy = 1$ Find $\dfrac{du}{dx}$ (06 Marks)

c. If $u = \dfrac{yz}{x}$, $v = \dfrac{xz}{y}$, $w = \dfrac{xy}{z}$, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. (07 Marks)

MODULE– III

5 a. Find div $\vec{F}$ and Curl $\vec{F}$ where $\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$ (07 Marks)

b. Using differentiation under integral sign,

Evaluate $\displaystyle\int_0^1 \frac{x^\alpha - 1}{\log x}\,dx \quad (\alpha \ge 0)$

Hence find $\displaystyle\int_0^1 \frac{x^3 - 1}{\log x}\,dx$ (06 Marks)

c. Trace the curve $y^2(a - x) = x^3, a > 0$ use general rules. (07 Marks)

OR

6 a. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ then prove that $\nabla r^n = n r^{n-2} \vec{r}$ (07 Marks)

b. Find the constants a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. Also find ϕ such that $\vec{F} = \nabla\phi$ (06 Marks)

c. Using differentiation under integral sign,

Evaluate $\int_0^{\infty} e^{-\alpha x} \frac{\sin x}{x} dx$ (07 Marks)

MODULE– IV

7 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$ (07 Marks)

b. Solve : $(1 + 2xy\cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$ (06 Marks)

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be temperature of the body after 40 minutes from the original? (07 Marks)

OR

8 a. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} \, dx$ (07 Marks)

b. Solve : $xy(1 + x y^2)\frac{dy}{dx} = 1$ (06 Marks)

c. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is parameter. (07 Marks)

MODULE– V

9 a. Solve by Gauss elimination method
$5x_1 + x_2 + x_3 + x_4 = 4$, $x_1 + 7x_2 + x_3 + x_4 = 12$, $x_1 + x_2 + 6x_3 + x_4 = -5$,
$x_1 + x_2 + x_3 + 4x_4 = -6$ (07 Marks)

b. Diagonalize the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ (06 Marks)

c. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by power method taking the initial eigen vector $(1, 1, 1)^1$ (07 Marks)

OR

10 a. Solve by L U decomposition method
$x + 5y + z = 14$, $2x + y + 3z = 14$, $3x + y + 4z = 17$ (07 Marks)

b. Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. (06 Marks)

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ into canonical form by orthogonal transformation. (07 Marks)

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